Scale Exploiting Minimal Solvers for Relative Pose with Calibrated Cameras

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Abstract

We present efficient minimal solvers for the estimation of relative pose between two calibrated images. The proposed method exploits known scale ratios of feature matches which are intrinsically provided by scale invariant feature detectors with scale-space pyramids (e.g. SIFT). Since we are using scale, the number of required feature matches is reduced and consequently fewer inliers are needed. In our paper, we derive two solvers each related to one of the two possible minimal sets: Our 1 + 3 point algorithm estimates relative pose from four feature matches with one known scale, while our 2 + 1 point algorithm computes relative pose from three feature matches with two known scales. We embed the proposed methods into RANSAC with two inlier classes, and present an efficient RANSAC modification for less reliable scales. Finally, we analyze performance and robustness in synthetic data, and evaluate our RANSAC approach on real data in comparison to RANSAC based on the five point algorithm.

1 Introduction

Relative pose estimation has been studied for centuries, first in the photographic and later in the computer vision community. Nevertheless, due to the fundamental relevance to many state-of-the-art applications in structure from motion and simultaneous localization and mapping, research is ever evolving in recent years. The core challenge of relative pose is to estimate the motion between two camera poses, revealing 3D reconstructions of the environment as well as camera odometry.

Classic approaches study the location of point correspondences in multiple views and their relationship to pose recovery [6, 8, 10, 16, 20]. In the case of unknown intrinsic calibration, the 8 point algorithm or 7 point algorithm is most frequently employed to recover the fundamental matrix [6, 8], and the 6 point algorithm is used for unknown focal length [20]. With calibrated cameras the 5 point algorithm is most suitable [16]. In our work we focus on the later situation where intrinsic camera parameters are available.

Recent literature tries to reduce the number of needed point correspondences by introducing additional hardware requirements or restrictive assumptions on the environment. In [10] a 4 point algorithm is introduced which requires known relative rotation angles between views, which may be provided by an inertial measuring unit, resulting in faster and more robust estimation of relative pose. Similarly, [14] propose a 3 point algorithm for efficient

Figure 1: We utilize intrinsically known scale from feature matching with scale-space pyramids (e.g. SIFT) to compute relative pose between calibrated images. Two minimal solvers are introduced: one for 4 correspondences with 1 known scale (green), and one for 3 matching features with 2 scales (red).

visual odometry with known rotation orientations. Finally, [10] exploits the setting of vehicle motion to estimate relative pose for self-driving cars using the locations of 2 point correspondences alone. In contrast, we present two new minimal solvers that reduce the needed number of correspondences with minimal additional overhead.

1.1 Contributions

Inspired by feature matching strategies such as SIFT [12] or ORB [18], which provide scale between correspondences, we introduce two solutions for relative pose with known scale ratios (fig. 1). In particular notice, as SIFT employs a scale-space pyramid, the patches’ scales on the image planes are intrinsically provided. We exploit the connection between scales and depth and derive geometric constraints linking pose and depth ratios. In total, there are two possible minimal sets for location and scale correspondences: We derive our 1 + 3 point algorithm which takes 4 feature matches with 1 known scale, and our 2 + 1 point algorithm which estimates relative pose from 3 matches with 2 known scales.

Our final estimation framework employs random sampling consensus (RANSAC) which is a popular estimation strategy frequently employed in estimation tasks affected by outliers [1, 2, 3, 16, 19]. We adapt RANSAC to our setup, and present an efficient modification for unstable scales. Finally we analyse the performance and robustness of our systems in comparison to the 5 point algorithm in [15]. In summary our contributions are:

1. We derive geometric observations relating scale (or depth) ratios to relative pose.
2. We exploit scale-space feature detection and derive two new minimal solvers for pose.
3. We embed our 1 + 3 and 2 + 1 point algorithm into RANSAC for unreliable scale.
4. We analyse performance and robustness in synthetic and natural images.

In our work we use the reciprocal relation between scale and depth, leading to a similar setup to the stereo system in [4]. In particular, [4] introduce a solver for pose between stereo cameras with minimal overlap. Due to the scale invariance of monocular images, their requirement of 4 points with 1 known depth value bares similarity to our 1 + 3 point algorithm. Contrasting [4], we establish a 1 + 3 and a 2 + 1 point algorithm for monocular views, and introduce a methodology to exploit the scale-space of common feature detectors.

Furthermore, we emphasize while [2] approximate the rotation matrix through linearization, we introduce an exact solver for the 1 + 3 setup.

Finally we note, while our method extracts scales using areas surrounding feature points, its mechanism is in stark contrast to patch-based approaches in literature [8, 13, 17]. Generally, these use affine patches or homographic frames from which additional point correspondences are created and applied to standard relative pose estimation techniques [8, 9, 13, 20]. However, a drawback with this approach is that an inaccurate local frame leads to incorrect points and consequently to incorrect results. None of these works propose frame refinement. In contrast, our method solely relies on robustly estimated location and scale using efficient refinement methods.

2 Geometric Observations

We denote N matched points in two views by homogenous vectors $\mathbf{x}_i = [x_i \ y_i \ 1]^T$ and $\mathbf{x}'_i = [x'_i \ y'_i \ 1]^T$, $i = 1, \dots, N$ respectively. Their corresponding 3D points in each frame's 3D coordinate system is found by $d_i \mathbf{x}_i$ or $d'_i \mathbf{x}'_i$, where d_i and d'_i denote the depth of points as observed from the relevant view. Furthermore, let us express the relative pose between images by rotation $\mathbf{R} \in SO(3) \subset \mathbb{R}^{3 \times 3}$ and translation $\mathbf{t} \in \mathbb{R}^3$. Without loss of generality, we assume the intrinsic calibration matrix $\mathbf{K}$ to be the identity. Now the epipolar constraint is given by

$$(\mathbf{x}'_i)^T \mathbf{E} \mathbf{x}_i = 0, \text{ with essential matrix } \mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}, \quad (1)$$

where the cross-product between $\mathbf{t}$ and any vector $\mathbf{a} \in \mathbb{R}^3$ is expressed by $[\mathbf{t}]_{\times} \mathbf{a} = \mathbf{t} \times \mathbf{a}$.

Given ratio $s_i = \frac{d_i}{d'_i}$ between depths d_i and d'_i of correspondence $(\mathbf{x}_i, \mathbf{x}'_i)$ from two views, we observe that the direction of the translation $\frac{\mathbf{t}}{|\mathbf{t}|}$ is determined by rotation $\mathbf{R}$, as

$$\mathbf{t} = d' \mathbf{x}' - d \mathbf{R} \mathbf{x} \quad \Leftrightarrow \quad \mathbf{t} = d'(\mathbf{x}' - s \mathbf{R} \mathbf{x}) \quad \Rightarrow \quad \mathbf{t} \propto \mathbf{x}' - s \mathbf{R} \mathbf{x}. \quad (2)$$

Therefore, with one known correspondence $(\mathbf{x}, \mathbf{x}', s)$ we rewrite (1) as

$$(\mathbf{x}'_i)^T (\mathbf{t} \times \mathbf{R} \mathbf{x}_i) = (\mathbf{x}'_i)^T ((\mathbf{x}' - s \mathbf{R} \mathbf{x}) \times \mathbf{R} \mathbf{x}_i) = (\mathbf{x}'_i \times \mathbf{x}')^T \mathbf{R} \mathbf{x}_i - (\mathbf{x}'_i)^T \mathbf{R} (s \mathbf{x} \times \mathbf{x}_i) = 0, \quad (3)$$

using equalities $\mathbf{a}^T \mathbf{b} = \mathbf{b}^T \mathbf{a}$, $\mathbf{a}^T (\mathbf{b} \times \mathbf{c}) = \mathbf{b}^T (\mathbf{c} \times \mathbf{a})$ and $\mathbf{R}(\mathbf{a} \times \mathbf{b}) = \mathbf{R} \mathbf{a} \times \mathbf{R} \mathbf{b}$ for $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$. Notice, rewriting (3) as the equality $(\mathbf{x}' - s \mathbf{R} \mathbf{x})^T (\mathbf{R} \mathbf{x}_i \times \mathbf{x}'_i) = 0$ reveals the intuition behind: all epipolar planes must contain the translation $\mathbf{t}$ or equivalently the vector $(\mathbf{x}' - s \mathbf{R} \mathbf{x})$.

Furthermore, (3) entails that all correspondences with depth ratios $(\mathbf{x}_i, \mathbf{x}'_i, s_i)$ satisfy

$$\mathbf{t} \times (\mathbf{x}'_i - s_i \mathbf{R} \mathbf{x}_i) = \mathbf{0} \quad \Leftrightarrow \quad \mathbf{t} \times \mathbf{x}'_i - \mathbf{t} \times s_i \mathbf{R} \mathbf{x}_i = \mathbf{0} \quad \Leftrightarrow \quad \mathbf{t} \times \mathbf{x}'_i = s_i \mathbf{t} \times \mathbf{R} \mathbf{x}_i, \quad (4)$$

where $\mathbf{a} \times \mathbf{b} = \mathbf{0} \Leftrightarrow \mathbf{a} \propto \mathbf{b}$ for $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$. Notice, (4) not only enforces the epipolar constraint though (3), it also ensures the sine rule of triangles holds. We employ (2), (3) and (4) to propose our minimal solvers in the following sections.

3 Proposed Minimal Solvers

We describe our approach for relative pose estimation between two calibrated monocular views. In contrast to many existing methods which use location alone, *e.g.* [8, 9, 13], we

exploit known scale ratios of feature matches. Our motivation comes from commonly employed feature detectors and descriptors, which rely on scale-space pyramid matching, *e.g.* SIFT[14], ORB[15]. Specifically, these methods intrinsically provide the scale difference between matched features in addition to the location information. We notice, known scale ratio on the image plane is closely linked to the depth ratio. In fact, the scale ratio is reciprocal to the depth ratio in homogeneous coordinate space:

$$\begin{bmatrix} x \\ s \\ y \\ s \\ 1 \end{bmatrix} \doteq \begin{bmatrix} x \\ y \\ s \end{bmatrix} \quad (5)$$

We denote this ratio between scales, or equivalently depth values, as $s_i = \frac{d_i}{d_j}$ in the following.

There are exactly two minimal sets which solve the relative pose problem under consideration of scale. In total we have 5 unknowns representing the transition between cameras, *i.e.* 3 for rotation and 3 for translation minus 1 as scale is irrecoverable. The 2D location of matching pixels provides one epipolar constraint. If scale is additionally provided, the sine rule in (4) must also hold. Henceforth there are two possible minimal solvers: (a) One point with scale and three points without ($1 \times 2 + 3 \times 1 = 5$), and (b) two points with scale and one point without ($2 \times 2 + 1 \times 1 = 5$). We introduce both in the subsequent sections.

3.1 1 + 3 Point Algorithm

Given a total of four matched points in the two image views, one with scale and location $(\mathbf{x}_1, \mathbf{x}'_1, s_1)$ and three with location alone $(\mathbf{x}_i, \mathbf{x}'_i), i = 2, \dots, 4$, we want to find rotation $\mathbf{R}$ and translation $\mathbf{t}$. Using (2) and (4), it is possible to express the following system of equations

$$\mathbf{x}'_1 - s_1 \mathbf{R} \mathbf{x}_1 = \mathbf{t} \quad (6)$$

$$(\mathbf{x}'_i \times \mathbf{x}'_1)^T \mathbf{R} \mathbf{x}_i - (\mathbf{x}'_i)^T \mathbf{R} (s_1 \mathbf{x}_1 \times \mathbf{x}_i) = 0, \quad i = 2, 3, 4. \quad (7)$$

We note however, (7) is independent of $\mathbf{t}$, and hence forms a system of 3 equations with 3 unknowns for rotation $\mathbf{R}$. We use this part as the core of our minimal solver, coined 1 + 3 point algorithm. Translation $\mathbf{t}$ is then found through (6), as length of $\mathbf{t}$ is irrecoverable.

We express rotation $\mathbf{R}$ as quaternion $q = q_a + q_b \mathbf{i} + q_c \mathbf{j} + q_d \mathbf{k}$, where

$$\mathbf{R} = 2 \begin{bmatrix} 0.5 - q_c q_c - q_d q_d & q_b q_c - q_d q_a & q_b q_d + q_c q_a \\ q_b q_c + q_d q_a & 0.5 - q_b q_b - q_d q_d & q_c q_d - q_b q_a \\ q_b q_d - q_c q_a & q_c q_d + q_b q_a & 0.5 - q_b q_b - q_c q_c \end{bmatrix}. \quad (8)$$

Eq. (7) can now be brought into a simple quadratic form of 3 equations (see supplement)

$$\mathbf{r}^T \mathbf{A}_i \mathbf{r} + \mathbf{b}_i \mathbf{r} + c_i = 0, \quad i = 2, 3, 4, \quad (9)$$

where the vector of unknowns corresponds to $\mathbf{r} = \begin{bmatrix} \frac{q_b}{q_a} & \frac{q_c}{q_a} & \frac{q_d}{q_a} \end{bmatrix}^T$, and $\mathbf{A}_i$, $\mathbf{b}_i$ and c_i are known observations. With rotation as homogenous quaternion, we derive a minimal solver through the Gröbner basis algorithm in [8]. Our system resolves into an 8 degree univariate polynomial with 8 solutions. Its roots can be found efficiently through Sturm sequencing [2] or more reliably through eigenvalue decomposition [8, 16].

Finally we emphasise, although (7) suffices as minimal solution, its naïve application to [8] leads to a 60% increase in non-zero monomials and steps needed to form the Gröbner basis. Consequently, our modified set of equations produces a numerically more stable solver, as fewer computational steps are necessary.

3.2 2 + 1 Point Algorithm

Let us now consider the minimal set with three point correspondences of which two have known scale, *i.e.* $(\mathbf{x}_1, \mathbf{x}'_1, s_1)$, $(\mathbf{x}_2, \mathbf{x}'_2, s_2)$ and $(\mathbf{x}_3, \mathbf{x}'_3)$. Using (2), (3) and (4), we start with the following system of equations

$$\mathbf{x}'_1 - s_1 \mathbf{R} \mathbf{x}_1 = \mathbf{t} \quad (10)$$

$$(\mathbf{x}'_1 - s_1 \mathbf{R} \mathbf{x}_1) \times (\mathbf{x}'_2 - s_2 \mathbf{R} \mathbf{x}_2) = \mathbf{0} \quad (11)$$

$$(\mathbf{x}'_3 \times \mathbf{x}'_1)^T \mathbf{R} \mathbf{x}_3 - (\mathbf{x}'_3)^T \mathbf{R} (s_1 \mathbf{x}_1 \times \mathbf{x}_2) = 0, \quad (12)$$

where (11) combines (4) with the proportional translation equivalence in (2). Similar to our 1 + 3 point algorithm, a system to solve for rotation $\mathbf{R}$ alone is given by (11) and (12), which consists of three independent equations. We simplify (11), by first rearranging it into

$$\mathbf{x}'_1 \times \mathbf{x}'_2 - s_1 \mathbf{R} \mathbf{x}_1 \times \mathbf{x}'_2 - s_2 \mathbf{x}'_1 \times \mathbf{R} \mathbf{x}_2 + s_1 s_2 \mathbf{R} (\mathbf{x}_1 \times \mathbf{x}_2) = \mathbf{0} \quad (13)$$

and then constructing a set of three equivalent quadratic equations, analogous to (9). When solving with Gröbner basis [8], we obtain a 4 degree univariate polynomial with 4 solutions. Again we find the roots through Sturm sequencing [9] or eigenvalue decomposition [8, 16].

3.3 Solutions and Degeneracies

In contrast to methods which find the essential matrix, our methods computes rotation $\mathbf{R}$ and translation $\mathbf{t}$ directly. Therefore, chirality is automatically enforced. However, as with other methods, our 1 + 3 and 2 + 1 point algorithm has degeneracies. If points lie on the same epipolar plane, the equations are not independent resulting in a degeneracy. Also, as we express rotation as homogeneous quaternion, camera motion with rotation angle 180° is unsupported. Both instances however are seldom experienced in real images.

4 Random Sampling Consensus Integration

RANSAC is a robust estimation framework which employs minimal solvers [1, 2, 3, 16]. In particular, minimal sets are randomly selected to find hypotheses which are then tested on the remaining data samples. Here, we first consider the two class setup, and then unstable scales to integrate our minimal solvers into the RANSAC framework.

4.1 Two Class RANSAC with Scale and Location

Our minimal solvers have two classes of sample points, specifically locations of point correspondences with scale and without. Therefore, similarly to [2], we modify the classical stopping criterion of RANSAC to reflect the two inlier types

$$K = \frac{\log(1-p)}{\log(1-r_s r_l^3)} \quad \text{or} \quad K = \frac{\log(1-p)}{\log(1-r_s^2 r_l)} \quad (14)$$

for the 1 + 3 and 2 + 1 point algorithm respectively. K is the number of iterations needed to guarantee an inlier set has been selected, up to a probability p , under the assumption that r_l and r_s represents the likelihood of selecting inliers for location and location with scale

Figure 2: The cameras are chosen with focal length 1 and random motion with baseline 0.1 and rotation of at most 45° . Points are sampled with depth values between 1 and 1.5.

respectively. Notice, in comparison to the 5 point algorithm, if $r_s < r_l^2$ fewer iterations are needed. The rates are based on inlier tests. Commonly, the Sampson distance is employed for estimating the error of the epipolar constraint for testing locations [24]:

$$\frac{((\mathbf{x}')^T \mathbf{K}_2^{-T} \mathbf{E} \mathbf{K}_1^{-1} \mathbf{x})^2}{(\mathbf{K}_2^{-T} \mathbf{E} \mathbf{K}_1^{-1} \mathbf{x})_1^2 + (\mathbf{K}_2^{-T} \mathbf{E} \mathbf{K}_1^{-1} \mathbf{x})_2^2 + (\mathbf{K}_1^{-T} \mathbf{E}^T \mathbf{K}_2^{-1} \mathbf{x}')_1^2 + (\mathbf{K}_1^{-T} \mathbf{E}^T \mathbf{K}_2^{-1} \mathbf{x}')_2^2} < \delta_l, \quad (15)$$

where $\mathbf{K}_1$ and $\mathbf{K}_2$ are calibration matrices of the first and second view respectively, $(\mathbf{a})_i^2$ computes the square of the i^{th} element of vector $\mathbf{a}$, and δ_l is a location inlier threshold.

We now establish an inlier test for scale. With (4), and under the assumption that $\mathbf{x}$ and $\mathbf{x}'$ are on the epipolar plane, we write

$$|\mathbf{t} \times \mathbf{x}'| = s |\mathbf{t} \times \mathbf{R} \mathbf{x}| \quad \Leftrightarrow \quad \frac{|\mathbf{t} \times \mathbf{x}'|}{|\mathbf{t} \times \mathbf{R} \mathbf{x}|} = s \quad \text{and inlier test} \quad \left| \frac{|\mathbf{t} \times \mathbf{x}'|}{|\mathbf{t} \times \mathbf{R} \mathbf{x}|} \frac{1}{s} - 1 \right| < \delta_s \quad (16)$$

as the normal vector of the cross-product is shared. The final term in (16) stems from the observation that scale from the image domain may differ from the expected scale implied by $\mathbf{R}$ and $\mathbf{t}$ which we denote $\hat{s}$ in the following. In particular, we define the scale error ε_s as the change required to match s with $\hat{s}$, such that the difference is given by $\hat{s} = s(1 + \varepsilon_s)$ leading to our inlier test above. Finally, we define as scale inliers the correspondences which satisfy the location constraint in (15) as well as the scale test in (16) up to small errors δ_l and δ_s . Notice, the magnitude of threshold δ_s can be easily established under consideration of parameters related to the step-size in the scale-space pyramid.

4.2 Efficient Bisection Algorithm for Unstable Scale

In high resolution images, feature detectors are fairly reliable for locations. Unfortunately, the estimation of scale is less stable and depends on the step-size used in the scale-space pyramid. For example, in SIFT scales are initially discretized by $s_l \in \{\sigma_0 2^{l\sigma_s} \mid l = 0, 1, 2, 3, \dots\}$ with base scale $\sigma_0 = 1.6$ and step-size $\sigma_s = \frac{1}{3}$, and then interpolated to continuous values [24]. Consequently, errors are inadvertently introduced which can affect our solvers.

We propose an efficient bisection search which is aimed at correcting errors in scale. In particular, we solve the relative pose problem iteratively with multiple scales taken around the original observation. Starting with solutions for s_i , $s_i(1 + \varepsilon)$ and $s_i(1 - \varepsilon)$, we update the scale with the best choice for s_i , half the variance $\varepsilon \leftarrow \frac{\varepsilon}{2}$, and continue the process. Let us emphasize, a monotonous performance measure with reducing error is required for bisections to work. We employ the location inlier ratio as performance measure, which we empirically show to be valid in our experiments in section 5.1.2.

Figure 3: Estimation with noisy locations (noise is standard deviation in CIF image pixel) for random motion. Mean E-error, t-error and R-error of 10,000 instantiations for our 1 + 3 and 2 + 1 point algorithms are compared to the 5 point algorithm.

Figure 4: Estimation with noisy scales (Gamma distributed around 1 with variance of one 100^{th} of SIFT's step-size) for random motion. Mean E-error, R-error and t-error of the 1 + 3 and 2 + 1 point algorithms are shown. (5 point algorithm is unaffected and hence not shown.)

Finally we note, as we employ bisection search, more solutions are required to be computed for the relative pose problem. The scoring of the varied scales is computationally the most significant cost, as in contrast our solvers are fast. Hence, preemptive methods, *e.g.* [14], and early bailout [8] should be employed for enhanced runtime performance.

5 Analysis and Experiments

We analyse robustness and accuracy of our proposed minimal solvers with synthetic data, and with real images as part of a RANSAC setup.

5.1 Performance Analysis in Noisy Conditions

In this section we employ a synthetic data model to evaluate our methods with known noise. In particular, we follow [14] to generate a challenging two view setup (fig. 2). Additionally we introduce noise on location and depth ratio observations in the following.

Our evaluation is based on three error measures:

$$\text{E-error:} \quad \min_i \left\| \frac{\hat{\mathbf{E}}}{|\hat{\mathbf{E}}|} - \frac{\mathbf{E}_i}{|\mathbf{E}_i|} \right\|_F \quad (17)$$

$$\text{t-error:} \quad \min_i \arccos \left(\frac{\hat{\mathbf{t}}^T \mathbf{t}_i}{|\hat{\mathbf{t}}| |\mathbf{t}_i|} \right) \quad (18)$$

$$\text{R-error:} \quad \min_i \arccos \left(\frac{\text{trace}(\hat{\mathbf{R}}^T \mathbf{R}_i) - 1}{2} \right) \quad (19)$$

where i indexes the set of solutions. The Frobenius norm between the true essential matrix $\hat{\mathbf{E}}$ and estimated essential matrices $\mathbf{E}_i$ in (17) is used to test overall system performance. Angular errors between true translation vector $\hat{\mathbf{t}}$ and estimations $\mathbf{t}_i$ or true rotation matrix $\hat{\mathbf{R}}$ and estimations $\mathbf{R}_i$, in (18) and (19) respectively, are employed for a more detailed analysis.

Figure 5: Scale effect on E-error (1st row) and the mean error on epipolar constraint for 10,000 inliers (2nd row) is shown for 10,000 trials with 2 + 1 (left, middle) and 1 + 3 point algorithm (right). The slice at center of 2 + 1 is shown with standard deviation, for comparison with 1 + 3. Both methods increase virtually monotonically with increasing noise.

5.1.1 Noise in Pixel Location

Fig. 3 shows our obtained errors with noise in location measurements. Here we induce normal distributed noise to the points on the image planes, which we denote as a standard deviation of pixel magnitude as observed by a CIF image (*i.e.* resolution $352px \times 240px$). Our 1 + 3 and 2 + 1 point algorithms (denoted 1 + 3pt and 2 + 1pt in the following) are compared to the five point algorithm in [16] which considers location alone (denoted 5pt). We modify E-error for 5pt to test $\pm \mathbf{E}_i$ and select the lowest. Similarly, we allow for $\pm \mathbf{t}_i$ and the two rotation matrices induced by $\pm \mathbf{E}_i$ with 5pt. In total, we generate 10,000 random instances of 3, 4 or 5 points using our data model, and we report on average performance.

Although 1 + 3pt only requires four locations in total, it is most vulnerable to noise, yet comparable to 5pt with similar results for t-error. 2 + 1pt performs significantly better than both other approaches, as the essential matrix derived is supported by two accurate scale measurements, and the solver is more stable.

5.1.2 Noise in Scale

We now have a closer look at 1 + 3pt and 2 + 1pt with noisy scale (5pt is unaffected by scale). Fig. 4 shows the average performance for 10,000 instantiations of different noise-levels (modelled as Gamma distribution around 1 with a variance of one 100^h of SIFT's default step-size).

Again, 2 + 1pt out-performs the 1 + 3pt setup, and rotations are especially improved. Notice, in (11) two scales must ensure that the rotation supports the translation, reducing the chance of incorrect results. In fact, we found that for large errors in scale, 2 + 1pt often produce few or even no real valued solution, which indicates some robustness to noise.

Finally fig. 5 models noise in scale systematically with increasing error. Notice, E-error for both, 1 + 3pt and 2 + 1pt, are virtually monotonously increasing with larger noise levels. This behaviour is essential for the bisection algorithm in section 4.2. We emphasize, although the true essential matrix is unknown in real data, an approximation of E-error is provided by estimating the inlier ratio, or alternatively the mean error in epipolar constraint.

Figure 6: Location ($\times$) and scale inlier ($\times$) for $2+1^*$ on image pairs p1 to p6 are shown. The ratio of location inliers r_l and scale inliers r_s is listed, and a subset of feature correspondences is shown. The histogram of scale divergence $\frac{\hat{\delta}_i}{s_i}$ with true scale $\hat{\delta}_i$ as estimated by $2+1^*$ is shown under the image pairs.

5.2 Real Data

We test the proposed methods on 6 real image pairs with varying degree of difficulty (fig. 6). First we extract SIFT feature matches using the standard setup in [10]. We then perform RANSAC (with $p = 0.99$, $\delta_l = 1px$, $\delta_s = 0.1$), and our evaluation is based on the location inlier criterion. The comparison is between the proposed methods and the five point algorithm in [16]. As RANSAC is based on a random selection process, we instantiate all algorithms 1,000 times and report averages. Note, real data is effected by both, noise in scale and location.

We compare the inlier ratio for 5pt, $1+3$ pt and $2+1$ pt in fig. 7. The performance without scale correction is significantly reduced for $1+3$ pt and $2+1$ pt – an indication that scale is much less reliable in real data than in our synthetic results. Fig. 7 additionally shows the results of $1+3$ pt and $2+1$ pt with bisection refinement (section 4.2), denoted $1+3^*$ and

Figure 7: Location based inlier ratio for 1,000 iterations of RANSAC is shown with standard deviation. The magenta line represents performance of post-refined 5 point algorithm using non-linear optimization on all inlier points.

Method	Iterations	Explored Solutions	Solver Runtime	Total Runtime
5pt	83	373	162ms	32s
1 + 3*	49	1920	18ms	103s
2 + 1*	33	3810	6ms	201s

Table 1: Average statistics of the different methods in RANSAC are listed using the naïve setting without early bailout. While the solver runtime for our methods is improved, the evaluation of the increased set of solutions (due to scale refinement) is costly in total runtime.

$2 + 1^*$. Overall these perform similarly to 5pt, with $2 + 1^*$ reaching the highest inlier ratios in p2, p3, p4 and p5, and $1 + 3^*$ having the best performance in p1 and p6. Fig. 6 shows visual examples for $2 + 1^*$. Image pair p1 has limited scale change and relatively simple camera configuration. Images p2, p3, p4 and p5 benefit from scale as we obtain reduced location dependence. A challenging setting for scale exploitation is p6, as the projection reduces the area of the patches’ ellipses consistently, resulting in incorrect scale ratios. This is also apparent in the divergence in scale, given by $\frac{\hat{s}_i}{s_i}$ with required scale $\hat{s}_i$ as estimated by $2 + 1^*$, which are normal distributions around 1 apart from results in p6 (fig. 6).

Tab. 1 shows detailed statistics of $1 + 3^*$, $2 + 1^*$ and 5pt within the RANSAC setting. Overall $1 + 3^*$ and $2 + 1^*$ improve upon RANSAC iterations and the minimal solver is much faster than that of 5pt (using the MATLAB solvers generated by [8]). However, in our scale refinement we instantiate the solver $1 + 2^3 = 9$ times for $1 + 3^*$ and $1 + (2 + 2)^3 = 65$ times for $2 + 1^*$ (with a greedy approach) leading to many explored solutions. Consequently, our proposed methods reduce in runtime. Here, early bailout [8] and preemptive evaluation [15] is advantageous and essential with less reliable scales.

6 Conclusion

Exploiting intrinsically provided scale ratios from scale-space feature matching techniques, we have introduced two minimal solvers for the minimal sets of relative pose estimation from scale and location correspondences. We then integrated our framework to RANSAC, using a two class inlier set and proposed a bisection search for refinement of less reliable scale. Finally, we present a performance analysis with synthetic data and evaluate our approach in comparison with the five point algorithm in a RANSAC setting.

Overall, our $1 + 3$ and $2 + 1$ point algorithms produce comparable results, but with reduced dependence on location inliers. The presented solvers are magnitudes faster than that of [16] in our experiments. However, with unreliable scale ratios, scale refinement is necessary, and we introduce an efficient bisection search for this. Early bailout [8] and preemptive evaluation [15] can further reduce runtime, and improved scale accuracy should be explored in future work.

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