Learning Robust Graph Regularisation for Subspace Clustering

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Abstract. Various subspace clustering methods have benefited from introducing a graph regularisation term in their objective functions [2]. In this work, we identify two critical limitations of the graph regularisation term employed in existing subspace clustering models and provide solutions for both of them. First, the squared l_2 -norm used in the existing term is replaced by a l_1 -norm term to make the regularisation term more robust against outlying data samples and noise. Solving l_1 optimisation problems is notoriously expensive and a new formulation and an efficient algorithm are provided to make our model tractable. Second, instead of assuming that the graph topology and weights are known a priori and fixed during learning, we propose to learn the graph [1] and integrate the graph learning into the proposed l_1 -norm graph regularised optimisation problem. Extensive experiments were conducted on five benchmark datasets.

Methodology. To address the aforementioned problems, we propose following objective function:

$$\min_{\mathbf{D}, \mathbf{Y}, \mathbf{W}} \quad \frac{1}{2} \|\mathbf{X} - \mathbf{D}\mathbf{Y}\|_F^2 + \lambda_1 \|\mathbf{Y}\|_1 + \lambda_2 \|\mathbf{Y}\mathbf{A}_\mathbf{W}\|_1 + \lambda_3 \|\mathbf{W}\|_F^2 \quad (1)$$

$$s.t. \quad \|\mathbf{d}_i\|^2 \leq 1, \mathbf{W}^T \mathbf{1} = 1, \mathbf{W} \geq 0.$$

where $\mathbf{X} \in \mathbb{R}^{r \times N}$ is a data matrix with N r -dimensional data feature vectors as columns, $\mathbf{D} \in \mathbb{R}^{r \times d}$ is a dictionary with d number of atoms, $\mathbf{W}$ is an affinity matrix that captures the topology of the data, $\mathbf{A}_\mathbf{W}$ is a matrix that is obtained by applying eigendecomposition on $\mathbf{W}$, and $\mathbf{Y} \in \mathbb{R}^{d \times N}$ is a sparse code matrix. In the following, we give explanation for each term:

(1) $\|\mathbf{X} - \mathbf{D}\mathbf{Y}\|_F^2$ is the reconstruction error term evaluating how well a linear combination of the atoms (columns) of the dictionary $\mathbf{D}$, can approximate the data matrix $\mathbf{X}$.

(2) $\lambda_1 \|\mathbf{Y}\|_1$ is a sparsity regularisation term on $\mathbf{Y}$, with a weighting factor λ_1 to favour a small number of atoms to be used for the reconstruction.

(3) $\lambda_2 \|\mathbf{Y}\mathbf{A}_\mathbf{W}\|_1$ is our proposed robust graph regularisation term. Note that we are using l_1 -norm instead of l_2 -norm weighted by λ_2 .

(4) $\lambda_3 \|\mathbf{Y}\mathbf{A}_\mathbf{W}\|_1 + \lambda_3 \|\mathbf{W}\|_F^2$ is the term with proper constraints ($\mathbf{W}^T \mathbf{1} = 1$ and $\mathbf{W} \geq 0$) for graph learning weighted by λ_2 and λ_3 .

The constraints, $\mathbf{W}^T \mathbf{1} = 1$ and $\mathbf{W} \geq 0$, are there to ensure the validity of the learned graph, while the constraint $\|\mathbf{d}_i\|^2 \leq 1$ ($\mathbf{d}_i$ is a column of $\mathbf{D}$ with $i = 1, \dots, r$) enforces the learned dictionary atoms to be compact.

Remark. Terms (3) and (4) are robust graph regularisation and graph learning terms, while first two terms, (1) and (2) constitute the conventional objective function of dictionary learning.

Optimisation. To solve the objective Eq. (1), we develop an algorithm based on ADMM.

Experiments. Table 1 shows experiments on benchmark datasets which are CMU-PIE (C-PIE for short), COIL, ORL, Yale, and YaleB. CA stands for clustering accuracy (%).

Table 1: Comparative results on C-PIE, COIL, ORL, Yale, and YaleB. ‘G’ stands for graph.

Methods	G	CA (%)				
		C-PIE	COIL	ORL	Yale	YaleB
l_1 G	No	70.3	67.1	66.8	40.0	48.4
SSC	No	72.1	58.9	55.5	38.7	52.6
LRR	No	71.5	45.1	66.2	46.2	65.0
LSR	No	77.9	56.0	56.5	48.5	62.4
CASS	No	82.6	59.1	68.3	45.6	81.9
GSC	Yes	100	80.9	61.5	43.4	74.2
R l_1 G	Yes	89.5	79.4	62.0	41.3	68.5
SMR	Yes	85.4	65.6	57.6	45.3	73.5
NSLRR	Yes	85.1	61.8	55.3	NA	NA
PCAN	Yes	82.5	76.5	49.1	54.4	59.2
LML	Yes	90.2	80.2	46.7	46.7	60.9
SDRAM	Yes	95.6	86.3	70.6	51.8	92.3
Ours	Yes	100	88.1	76.3	59.6	95.2

References

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