This work introduces a new statistical model of visibility in point-sampled scenes, such as those constructed from multiple lidar or depth-camera scans. A visibility density is defined along each optical ray, which gives the probability that a non-occluded scene-point exists at any particular location, with respect to a given camera. The new approach avoids any commitment to a surface-mesh, in order to develop a more data-driven probabilistic model. Furthermore, this approach naturally allows for the existence of gaps and uncertainty in the point-cloud (see fig. 2).

The new model has potential applications to multi-view stereo problems, in which visibility is an essential component of the photometric re-projection error [3]. There are other potential applications to the graphical rendering of point-cloud data [2].

If the scene consisted of randomly distributed particles, of radius \( \varepsilon \), then it could be modelled as a volumetric Poisson distribution. It would follow that the probability of point \( p(t) \) being non-occluded would be \( \text{pr}(\hat{\theta}_C) = \exp(-\lambda |C|) \), where \( |C| = \pi \varepsilon^2 t \) is the volume of a cylinder, which must be empty, between \( p(t) \) and the optical centre. This can be generalized to the case of non-uniformly distributed points, leading to a vacancy density corresponding to (1) and the blue polygons in fig. 1:

\[
\text{pr}(\hat{\theta}_C | R, S) = \exp(-\eta \Lambda(t))
\]

where \( \eta \) is a free parameter, and \( \Lambda(t) \) is a generalized volume, which increases in dense regions of the point cloud. This can be defined as:

\[
\Lambda(t) = \int_0^t \text{pr}(\hat{\theta}_C | R, S) \, dt.
\]

It can be shown that \( \Lambda(t) \) is a product of generalized step-functions. These steps are probabilistic representations of potential occluders, lying between \( p(t) \) and the optical centre.

Substituting (4) and (5) into (3) gives the final visibility density, corresponding to the green curves in fig. 1, along ray \( R \):

\[
\text{pr}(\hat{\theta}_C | R, S) = \frac{\exp(-\eta \Lambda(t))}{|R|/S} \sum_k w_k G((t - \mu_k)^2 / \sigma_k^2). 
\]

The scalar \(|R|/S|\) is a normalizing constant, representing the ‘total intersection’ of the ray with the probabilistic scene-model, which can be computed numerically.

The model was evaluated by computing reference visibilities in high-resolution point-clouds [2], then decimating these clouds, and re-estimating the visibility of a large number of test-points. An ROC analysis was performed on the estimated vs. true states (visible/occluded) of the test points. The results indicate that the new model outperforms naive visibility tests, in addition to the theoretical contributions outlined here.

