Variational Level Set Segmentation in Riemannian Sobolev Spaces

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The variational level set method [9] is still one of the most widely used methods in computer vision – especially for image segmentation [1]. This popularity might seem surprising, because variational level set segmentation is known to be non-convex, e.g., [3]. All the more, because since the seminal work of Chan et al. [3] a lot of research has been carried out in order to develop efficient methods for solving convex models for image segmentation, cf. [1, 2, 5].

The non-convexity of the variational level set approach is caused by the usage of continuous but non-convex approximations of the Heaviside and Dirac distribution for defining area and boundary integrals. This non-convexity is, however, not always a bane, because variational level set formulations for localized active contours models [6] or image segmentation in the presence of intensity inhomogeneities [7] make extensive usage of smeared-out Heaviside and Dirac distributions. As a consequence, it is still of interest to develop efficient methods for the non-convex variational level set method for image segmentation, which is the goal of this paper. Thereby, we will consider so-called Sobolev gradient flows, which have recently been shown to be superior to classical L^2-based gradient flows [4, 8]. Inspired by [10], we extend these approaches by changing the notion of distance in H^1. The main observation which leads to the proposed approach is that standard gradient for variational level set segmentation take the form

$$\nabla\mathcal{E}(\phi(x)) = F(x, I(x), \phi(x), \nabla\phi(x)), \tag{1}$$

where $\mathcal{E}$ is the energy to be minimized, I denotes the image to be segmented, and ϕ is the level set function. As a consequence, the gradient does not only inherit the very local behavior of the image, making the resulting level set evolution prone to get stuck in local minima, but also varies significantly w.r.t. to the individual problem dimensions, i.e., pixels. Both of the issues can be cure with the proposed approach which essentially projects this gradient into a Sobolev space endowed with a carefully chosen inner product. Thus, the minimizing gradient flow in the Riemannian Sobolev space exhibits a significantly improved convergence, compared to gradient flow in H^1. This advantage in convergence translates directly to an improvement of the overall runtime, cf. Fig. 1.

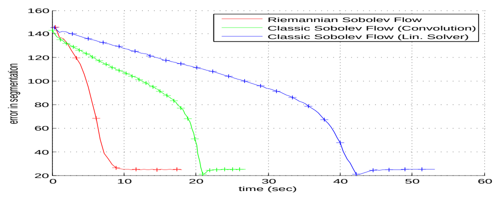
$$\boxed{\langle\phi,\psi\rangle_{H^1} = \langle\phi,\psi\rangle_{L^2} + \alpha\,\langle\nabla\phi,\nabla\psi\rangle_{L^2}}$$

$$\Downarrow$$

$$\boxed{\langle\phi,\psi\rangle_{M} = \langle\phi,\psi\rangle_{M_0} + \langle\nabla\phi,\nabla\psi\rangle_{M_1}}$$

(a)

(b)



(c)

Figure 1: **The proposed generalization** (a) results in efficient Riemannian Sobolev flows, which provide accurate results (b), however with significantly improved convergence and overall runtime (c). Every 5th iteration is marked with a $+$.

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[1] This fact is proven by 579 hits since 2012 obtained by a Google scholar search for the query "'variational level set' + 'segmentation'" performed on 9th of May 2014.