Throughout the literature the calibration quality obtained by different authors, even when using comparable techniques, is somewhat fluctuating. Given the complex nature of the image acquisition process and the uncertainties of the target manufacturing, it is often difficult to identify the exact sources of error and how they combine. The quality measure itself, which is usually the reprojection error, is not always a correct indicator. In [1] Douxchamps suggests that a higher accuracy could be attained by increasing the total interface length of the markers themselves and thus enhance their localization. A very in-depth theoretical analysis is validated by a large amount of experiments with synthetic images; however, real-world tests exhibit errors about an order of magnitude larger, which the author explains in part with the non-planarity of the target and in part with optical artifacts not corrected by the calibration model. Similar differences between results obtained with synthetic and real images have been previously observed by Heikkilä in [2].

While we agree that this discrepancy can be attributed to imperfection of the target manufacturing, we argue that the non-planarity is not the main error source. Specifically, we built a planar target by stitching a checkerboard printed on inextensible plotter paper onto a 6mm thick highly planar float glass. Using this target we intrinsically calibrated a test camera with the Zhang method and, beside studying the reprojection error with respect to image points, we also plotted the same oriented error bars applied to the theoretical model of the checkerboard. The results of this experiment are shown in Fig. 1. It is immediate to observe that the measurements are not subject to an uniformly distributed error. From a general point of view this means that the optimization was not able to fit the theoretical checkerboard model with a zero-mean Gaussian error, which in turn highlights the presence of some systematic error source. Basically, this source can be correlated with three causes: a localization bias introduced by the subpixel corner detector, the inability of the adopted camera model to fully capture the real image formation model (namely lens distortions), or some unknown discrepancy between the theoretical checkerboard model and the printed one. We are not inclined to think that the bias in corner detection is significant since, if this was the case, we should spot a higher coherence in error orientations as all the corner features present roughly the same orientation, scale and illumination conditions. To rule out the deficiency of the camera model we plotted the same error measurements on the image plane space (i.e., the camera CCD sensor). By observing part (b) of Fig. 1 it is quite evident that the error distribution is isotropic and this is a strong indication that no systematic error is amendable by using a more sophisticated camera model. Those considerations suggest that the printing process itself could be inaccurate enough to represent a significant source of systematic error in the calibration process. For this reason, we decided to investigate the nature of such printing errors and to tailor a calibration procedure able to correct them. It should be noted that the influence of printing error was already observed in literature. Recently Strobl [4] described a similar scenario, albeit he suggested that the printing procedure introduces just two types of systematic biases: an error in the global scale of the checkerboard and a non predictable aspect ratio. While this simple formulation allows for an elegant calibration procedure that introduces just two additional parameters, we think that it is not general enough. In fact, the distribution of the error shown in Fig. 1 can not be justified by scale and aspect ratio transformations.

In order to deal with the most general scenario we propose to lift any direct constraint on the target geometry and iteratively run a three-step procedure. In the first step we assume to know precisely the calibration target and we perform a standard calibration procedure with the currently trustable target model. Once plausible camera parameters are obtained, we assume them to be correct and we evaluate a more accurate target geometry by using a bundle adjustment technique to estimate only camera poses and scene (i.e. target). At this point a further step is needed, specifically we need to rescale the newly created target to fit the original one. This is necessary since the bundle adjustment step does not guarantee scale invariance. This adjustment step is performed by using the robust closed form point set alignment technique by Horn [3]. The procedure is stopped when the reprojection error falls below a fixed threshold or a maximum number of iterations is reached. Note that since the estimated target is scaled towards the theoretical one at each step, the final camera calibration could be subject to an absolute scaling error that is not avoidable as the real measures of the target are not known. Still, this error is averaged over the printing error of each corner and in practice this has shown to be very small (see Fig. 2).

Figure 1: The strongly polarized distribution of the reprojection error with respect to target points obtained in our calibration experiments (a) suggests that the measurements on images were not subject to zero-mean error. On the other hand, the distribution of the error on the image plane (b) is isotropic, assessing the good compensation of radial and tangential distortions and the reliability of the corner detector. This supports our hypothesis that the discrepancies are due to systematic printing errors (errors magnified by a factor of 100).

Figure 2: Reprojection error (magnified by a factor 100) with respect to the model points (a) and the sensor plane (b) after our method comes to convergence.