This article deals with a simple and flexible extrinsic calibration method, for non-overlapping camera rig. The aim is the calibration of non-overlapping cameras embedded on a vehicle, for visual navigation purpose in urban environment. The cameras do not see the same area at the same time. The calibration procedure consists in manoeuvring the vehicle while each camera observes a static scene. The main contributions are a study of the singular motions and a specific bundle adjustment which both reconstructs the scene and calibrates the cameras. Solutions to handle the singular configurations, such as planar motions, are exposed. The proposed approach has been validated with synthetic and real data.

This article can be seen either as an improvement of the algorithm proposed by Esquivel et al. [1], or as an extension of the classical bundle adjustment proposed by Triggs et al. [2] for rigidly linked cameras (with possible totally non-overlapping fields of view) for a calibration purpose.

We consider at least \( N_{cam} \geq 2 \) rigidly linked cameras, with known intrinsic parameters and non-overlapping fields of view. The cameras \( C_i \) are assumed to be synchronized. \( K \) motions are performed. Thus, \( K + 1 \) is the number of poses of the multi-camera rig over the time. \( C_i^k \) represents the \( k^{th} \) camera at time \( k \). Each pose of the camera \( C_i^k \) is expressed relative to its first pose at time \( k = 0 \). Let \( T_i^k \) be the homogenous transformation of \( C_i^k \) coordinate system from time 0 to time \( k \), for \( C_i \in \{1...N_{cam}\} \) (see Figure 1). In the same way, \( \Delta T_i \) is the unknown homogeneous transformation from \( C_i \) to \( C_{i+1} \) coordinate system.

While the camera rig is moving, each camera \( C_i \) observes a static scene \( S_i \) of 3D points. \( s_i^k \) are the projected points of the scene \( S_i \), in the image plane of \( C_i^k \).

The calibration (see Figure 2) consists first in computing each camera trajectory \( T_i^k \). Second, the extrinsic parameters \( \Delta T_i \) are linearly initialized, and finally refined as well as the scenes thanks to a specific bundle adjustment.

We introduce a linear initialization of the extrinsic parameters for general or singular motions. An estimate of the relative pose \( \Delta T_i \) can be obtained linearly from the trajectory \( T_i^k \) of each camera. The rigidity assumption and simple changes of basis lead to:

\[
\forall i \in [1...N_{cam}], \forall k \in [1...K], \quad T_i^k \Delta T_i = \Delta T_i T_i^k
\]  

We solve the equation (1) for general motions and we outlines the critical motions where the equation (1) cannot be solved in the same way. The singular motions are rotations and screw motions about an axis (when axes of rotation are the same), or pure translations, planar motions and screw motions with parallel axes (when axes of rotation are different). For these singular motions, the calibration is partial: only some parameters can be recovered.

The article focuses on planar motions — the most usual singular case for a mobile robot (see Figure 3). An initial estimate of the extrinsic rotations is provided, then we show a practical way to initialize the camera relative heights. Lastly, the specific bundle adjustment is applied.

A specific bundle adjustment is used to optimize the extrinsic parameters, the scene and the trajectory of the multi-cameras rig. The algorithm is initialized with the linear estimate of the extrinsic parameters and the union of all scene points, expressed in the same coordinate system. Let \( M \) be the number of all the 3D points. Finally, \( 6(K+1)+6(N_{cam}-1)+3M \) parameters are optimized by a Levenberg Marquardt algorithm during the minimization of the reprojection errors.

The results validate our approach with both synthetic and real data. First, the accuracy of the proposed calibration method is demonstrated with an overlapping stereo camera system (Figure 4a). Second, the feasibility of the approach is illustrated with non-overlapping cameras embedded on a vehicle (Figure 4c).

![Figure 4: Calibration of stereo camera with non-overlapping assumption, for overlapping cameras (a) and (b) and embedded non-overlapping cameras (c).](image)

Even if the motions are planar, the scene permutation solution allows to get a full calibration. Moreover, the results show that our calibration, with non-overlapping assumption, is as accurate as classical algorithm with overlapping fields of view (below 0.5 mm for the translation and about 0.01° for the rotation).
