Two View Geometry Estimation with Outliers

Olof Enqvist
http://www.maths.lth.se/vision/people/olofe
Fredrik Kahl
http://www.maths.lth.se/vision/people/fredrik

Centre for Mathematical Sciences
Lund University
Sweden

Estimating the relative orientation of two cameras is a classical problem in vision. Probably the most well-known method is the eight-point algorithm introduced by Longuet-Higgins in 1981 [5], and modified by Hartley [3] to include normalization. Although normalization made the algorithm more robust, there are still algorithmic degeneracies and the algorithm breaks down in the presence of outliers.

For calibrated cameras a guaranteed optimal algorithm minimizing reprojection errors appeared just recently in [2] but it cannot handle outliers in the correspondence set. Thus, heuristic methods like hypothesize-and-test approaches are normally used to remove outliers, [1, 4]. In this paper we address the problem of uncertain correspondences by formulating it as a mathematical optimization problem. For calibrated cameras, we provide both necessary and sufficient geometric constraints for an optimal solution. Based on this analysis, we propose an algorithm to find the optimal set of correspondences as well as the optimal relative orientation.

To make this more precise, we say that a correspondence $(x,\bar{x})$ is *consistent* with a relative orientation (R,t) if there exists a 3D point X such that its angular reprojection errors satisfy

$$\angle(x,X) < \varepsilon \quad \text{and} \quad \angle(\bar{x},R(X-t)) < \varepsilon. \tag{1}$$

Here ε is a predefined error tolerance. We seek a relative orientation which is consistent with as many correspondences as possible.

Our approach is based on a branch and bound search over the possible relative orientations. The following table gives an overview. First of all we need to decide on an error tolerance and some lower bound on the number of inliers of the optimal solution.

Iterate until desired precision is reached:
1. Pick a box from the queue.
2. Try to detect and remove outliers.
3. Try to discard the box.
4. If the box cannot be discarded:
   - Divide the box and update the queue.
   - Try improve the bound on the optimum.
5. Remove the box from the queue.

We will now look closer at some of the elements of this algorithm. In order to get a simple constraints, we use a rather different parameterization of the relative orientation. We introduce new bases in both cameras with coinciding z-axes parallel to the epipole of camera 1 and define α to be the relative rotation angle of these bases. Specifying these z-axes in the local camera coordinate systems and specifying the angle α is one way to define a relative orientation.

To introduce the constraints, we assume for a moment that there is no noise. Consider the new coordinate systems introduced above and corresponding spherical coordinates for the image points. For a consistent correspondence, the difference between the azimuthal angles measured in the first and second image will satisfy $\varphi = \bar{\varphi} + \alpha$. It turns out that we get simpler constraints if we choose to look at pairs of correspondences. Naturally,

$$\varphi_j - \varphi_k = \bar{\varphi}_j - \bar{\varphi}_k. \tag{2}$$

One way to view this is as the angle between epipolar planes, see Figure 1. We will denote these angles $\gamma_{jk} = \varphi_j - \varphi_k$ and $\bar{\gamma}_{jk}$. Furthermore, to get positive depths we require the polar angles to satisfy,

$$\theta \geq \bar{\theta}. \tag{3}$$

Now, if we allow reprojection errors these relations are no longer exact, but the extra uncertainty can be handled and it can be shown that the constraints are still sufficient for consistency. This is stated in Theorem 2 in the paper.

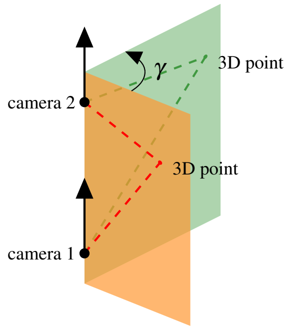

Figure 1: The angle $\gamma = \varphi_j - \varphi_k$ between two epipolar planes.

So how do we handle outliers? Regarding the constraint on the polar angles it is simple enough. A correspondence which does not satisfy this constraint must be an outlier. For the pairwise constraints on φ it is more complicated. A violated constraint only implies that one of the involved correspondences is incorrect, but we need to determine which one. It turns out that the problem can be solved by finding the minimal vertex cover for an undirected graph. This is often referred to as the vertex cover problem and there are many algorithms available, but for our purposes rather simple approximations will be sufficient.

Returning to the main algorithm, note that we parameterized the relative orientation using two unit vectors and a rotation angle. To find the optimal values for these parameters we perform a branch-and-bound search over the two vectors. The possible values thus lie in $\mathbb{S}^2 \times \mathbb{S}^2$ and we need some method to search this space. We use a branch and bound approach dividing the spheres into smaller and smaller spherical triangles. To check the pairwise constraints, we need bounds on the possible values of the azimuthal angles. Our first observation is that it is sufficient to consider the boundaries of the spherical triangles. This is Theorem 3 of the paper.

There is an apparent problem with this approach. The number of constraints is quadratic in the number of correspondences, since we are considering all pairs. To address this we compute simple under- and over-estimators for φ_i. These can then be used to produce under- and overestimators for the γ_{jk}'s in the first image and similarly for $\bar{\gamma}_{jk}$ in the second image. This way most work is performed on the φ_j's making it linear in the number of image points.

To our knowledge, the proposed algorithm is the first that is guaranteed to find the optimal relative orientation in the presence of outliers. Theoretic discussions as well as experiments demonstrate its ability to compute the globally optimal solution while discarding outliers. Execution times are still high for real-time applications but we believe that a significant speedup is possible by saving results that can be re-used and working on the function approximations.

[1] M. A. Fischler and R. C. Bolles. Random sample consensus: a paradigm for model fitting with application to image analysis and automated cartography. *Commun. Assoc. Comp. Mach.*, 24:381–395, 1981.

[2] R. Hartley and F. Kahl. Global optimization through rotation space search. *Int. Journal Computer Vision*, 82(1):64–79, 2009.

[3] R. I. Hartley. In defense of the eight-point algorithm. *IEEE Trans. Pattern Analysis and Machine Intelligence*, 19(6):580–593, 1997.

[4] R. I. Hartley and A. Zisserman. *Multiple View Geometry in Computer Vision*. Cambridge University Press, 2004. Second Edition.

[5] H. C. Longuet-Higgins. A computer algorithm for reconstructing a scene from two projections. *Nature*, 293:133–135, 1981.