A Comparative Analysis of Radial-Tchebichef Moments and Zernike Moments

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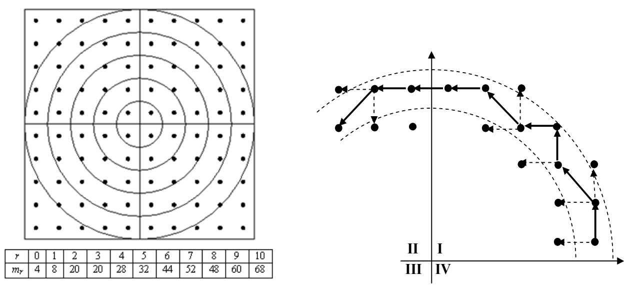
Orthogonal moment descriptors are commonly used in applications such as image classification, pattern recognition and identification. A radial-polar representation of image coordinate space is particularly useful in the above applications, since it facilitates the derivation of rotation invariants of any arbitrary order. Zernike moments and radial-Tchebichef moments fall into the category of moments that are defined using radial-polar coordinates. The discrete orthogonal nature of the kernel of radial-Tchebichef moments provides notable advantages over continuous Zernike moments. The paper presents a detailed analysis to prove that radial-Tchebichef moments have superior features compared to Zernike moments and are computationally less complex. This paper also presents a novel framework for accurately computing moments with kernel defined using polar coordinates, that is particularly suitable for discrete orthogonal rotation invariants. The method preserves the separability property of the kernel, which can be effectively used in computing both forward and inverse moment transforms. Thus the proposed framework yields a simple and fast implementation of radial-Tchebichef moments for image reconstruction, and invariants for pattern recognition. The efficiency of the proposed method is demonstrated through a series of experimental results.

The most appropriate mathematical structure for computing radial Tchebichef moments is a set of discrete concentric rings, where each ring represents a fixed integer value of radial distance r from the centre of the image. We can subdivide the coordinate space into $N/2$ concentric rings R_r , $r = 0,1,\ldots,(N/2)-1$. Depending on the number of points inside a ring, each ring can be further subdivided into m_r regions. An example for $N = 10$ is shown in Figure 1.
On ring R_r, the angle θ varies from 0 to 2π in m_r discrete intervals such that

$$\theta_k = \frac{2\pi k}{m_r}, \quad k = 0, 1, 2,\ldots, m_r-1. \tag{1}$$

Eq. (1) is valid only under the assumption that there exists a one-to-one mapping between pixels on the image, and points that are distributed uniformly around concentric circles. If we denote the image intensity value at location (r, θ_k) by $f(r, k)$, then the radial-Tchebichef moments of order p and repetition q are defined using the equation

$$T_{pq} = \sum_{r=0}^{N/2-1} t_p(r) \sum_{k=0}^{m_r-1} \frac{1}{m_r} e^{-j\frac{2\pi qk}{m_r}} f(r,k),$$
$$p = 0, 1, \ldots, (N/2)-1, \quad q = 0,1,\ldots,m_r-1. \tag{2}$$



r	0	1	2	3	4	5	6	7	8	9	10
m_r	4	8	20	20	28	32	44	52	48	60	68

(a) (b)

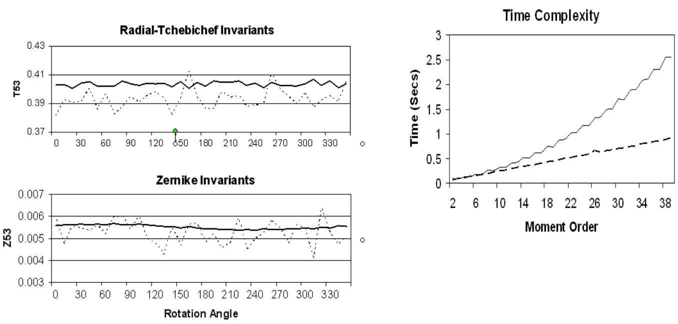
Figure 1: (a) A 10x10 pixel space subdivided into 5 concentric rings $R_0..R_4$. The middle ring R_0 contains 4 pixels whereas the ring R_4 contains 28 pixels. (b) The ring traversal algorithm for ring R_4 starts with the lowermost point within the ring in the first quadrant, and incrementally visits all the remaining points in the ring. The arrows point to the candidate pixels at each iteration, and the solid arrow points to the next selected pixel.

The ring traversal algorithm is very efficient in that it visits each pixel in the image exactly once, and yields a one-to-one mapping of Cartesian coordinates of pixels to radial-polar coordinates. If m_r denotes the number of pixels on ring R_r, we can calculate the following one-dimensional moments using circular functions for each r:

$$\alpha_q(r) = \frac{1}{m_r}\sum_{k=0}^{m_r-1}\cos\left(\frac{2\pi q\,k}{m_r}\right) f(r,k)$$
$$\beta_q(r) = \frac{1}{m_r}\sum_{k=0}^{m_r-1}\sin\left(\frac{2\pi q\,k}{m_r}\right) f(r,k), \quad r = 0 \ldots N/2-1, \quad q = 0\ldots m_r-1. \tag{3}$$

Both the above functions are periodic in q, and therefore, for a given r, the value of q need be varied from 0 to m_r-1 only. This result yields an efficient procedure for the computation of radial-Tchebichef invariants.

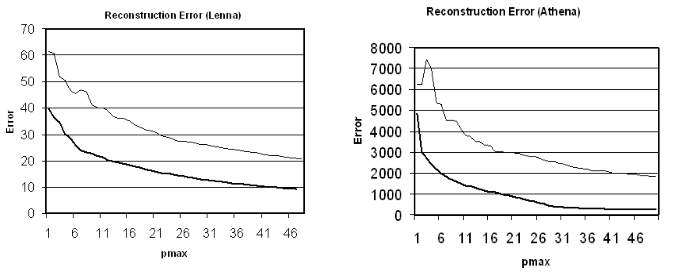
For the analysis of invariant characteristics of Zernike and radial-Tchebichef moments, different types different types of binary and gray-level images were used. The values of Zernike moments tend to become very small as the moment order increases. Radial-Tchebichef moments showed consistently good invariant characteristics for high-order moments (p, q >4). The analysis of rotation invariance of computed moments was repeated by adding 2% salt-pepper noise to the rotated images. The plots of invariants computed using radial-Tchebichef moments and Zernike moments for the binary image are given in Figure 2(a).
Figure 2(b) shows the variation of CPU time with moment order for Zernike and radial-Tchebichef moments of a 256x256 gray-level image, as obtained on a 2.8GHz processor with 1GB RAM.



(a) (b)

Figure 2: (a) The solid line gives the plot of invariants without noise, and the dotted line gives the values computed using the noisy image. (b) The solid line gives the computational complexity of Zernike moments, while the dotted line gives the complexity of Raidal-Tchebichef moments. (4)

The reconstruction accuracy of radial-Tchebichef moments was also analysed using both binary and gray-level images. The ring traversal algorithm gave very good reconstructions, owing to the one-one correspondence between pixel values and the intensity values along each circular ring. A comparison of the magnitudes and variations of reconstruction errors with moment order given in Figures 3(a), 3(b) clearly show the superior feature representation capability of radial-Tchebichef moments over Zernike moments.



(a) (b)

Figure 3: Plot of reconstruction errors with respect to moment order, (a) for the gray-level image "Lenna" and (b) for the binary image "Athena". The thin line on the top is obtained using Zernike moments, and the thick line using Radial-Tchebichef moments.