Minimal Solutions for Panoramic Stitching with Radial Distortion

Martin Byröd[1]
byrod@maths.lth.se
Matthew Brown[2]
mbrown@cs.ubc.ca
Kalle Åström[1]
kalle@maths.lth.se

[1] Centre for Mathematical Sciences
Lund University
Lund, Sweden

[2] Dept of Computer Science
University of British Columbia
Vancouver, Canada

This paper presents a solution to panoramic image stitching of two images with coinciding optical centers, but unknown focal length and radial distortion. The algorithm operates with a minimal set of corresponding points (three) which means that it is well suited for use in any RANSAC style algorithm for simultaneous estimation of geometry and outlier rejection. Compared to a previous method for this problem, we are able to guarantee that the right solution is found in all cases. The solution is obtained by solving a small system of polynomial equations.

The problem of dealing with cameras with various forms of non-linear distortions in computer vision is not new, but traditionally these lens effects have only been incorporated at the bundle adjustment stage. However, in situations with more than very mild distortions, it might be necessary to account for non-linear effects already at the RANSAC stage. Recently a number of contributions have been made on the efficient estimation of geometric relations in computer vision along with radial distortion [1, 4].

A direct inspiration for this work is the two-point algorithm for estimating rotation and focal length by Brown *et al.* [2]. This algorithm does however not handle any distortion and we show that for non-standard lenses, this might be insufficient.

Most closely related to our approach is the work by Jin [6]. Jin formulates the same problem as we do. However, Jin uses an iterative optimization based scheme which is not guaranteed to find the right solution.

We consider a setup with two cameras P_1 and P_2 with a common focal point. We fix a coordinate system where the common focal point coincides with the origin and such that the first 3x3 part of the matrix P_1 is the identity. Moreover, we have a set of world points $\{X_j\}$ and corresponding image projections $\{u_{1j}\}$ and $\{u_{2j}\}$. We assume square pixels, zero skew and centered principal point [5] and thus obtain the following relations

$$\lambda_{1j} u_{1j} = K X_j, \quad \lambda_{2j} u_{2j} = K R X_j, \tag{1}$$

where $K = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix}$, R is a rotation matrix and the λs are the depths. By normalizing to remove the dependence on λ_{ij} and solving for X_j we can write down the constraints

$$\frac{\langle K^{-1}u_{1j}, K^{-1}u_{1k}\rangle^2}{|K^{-1}u_{1j}|^2 |K^{-1}u_{1k}|^2} = \frac{\langle X_j, X_k\rangle^2}{|X_j|^2|X_k|^2} = \frac{\langle K^{-1}u_{2j}, K^{-1}u_{2k}\rangle^2}{|K^{-1}u_{2j}|^2 |K^{-1}u_{2k}|^2}. \tag{2}$$

In the above equation f only occurs in even powers and hence we set $p = f^2$. Moreover we multiply through with p^2 to remove any $1/p^2$ terms. Finally we multiply up the denominators which makes the 4th degree terms cancel out leaving a 3rd degree polynomial in p. This formulation was used in [2] to solve for the focal length.

Now let x denote measured image coordinates affected by radial distortion and let u denote the corresponding pin-hole coordinates. We model radial distortion using Fitzgibbon's division model

$$|x| = (1 + \lambda |x|^2)|u|, \tag{3}$$

where $|\cdot|$ is the vector length and λ is the radial distortion coefficient. Allowing us to write

$$u \sim x + \lambda z, \tag{4}$$

where $z = [\ 0 \quad 0 \quad x_1^2 + x_2^2\ ]^T$.

We now insert (4) into (2) and obtain a polynomial of degree 3 in p and degree 6 in λ (the 8th and 7th degree terms in λ cancel out). The minimal setup for solving this system is three point correspondences and the theoretical number of solutions is 18.

To solve the system, we make use of Gröbner basis techniques [3, 7], which in this case computationally amounts to an LU decomposition of a 90×132 matrix and an eigenvalue decomposition of an 18×18 matrix with a running time of 13 milliseconds/instance in our implementation (on a 2 *Ghz* machine). The code is available for download at http://www.maths.lth.se/vision/downloads.

The conclusion is that including radial distortion at the RANSAC stage is beneficial compared to distortion free approaches in terms of number of inliers found and overall precision. A particular advantage is the ability to recover inliers evenly over the whole image where an algorithm which does not model distortion will only keep point matches close to the centers of the images. Radial distortion occurs frequently in cheap consumer cameras as well as in high-end lenses depending on the type of lens. Thus this work should be interesting to consider for any image stitching system.

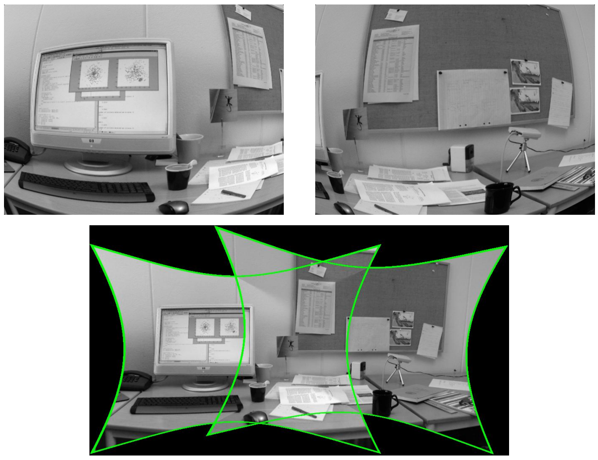

Figure 1: Left: Two images with heavy radial distortion taken with a common focal point. Right: The same two images after rectification and alignment using the stitching pipeline presented in this paper.

[1] J. Barreto and K. Daniilidis. Fundamental matrix for cameras with radial distortion. In *IEEE International Conference on Computer Vision*, Beijing, China, 2005.

[2] M. Brown, R. Hartley, and D. Nister. Minimal solutions for panoramic stitching. In *Proceedings of the International Conference on Computer Vision and Pattern Recognition (CVPR07)*, Minneapolis, June 2007.

[3] M. Byröd. Fast and stable polynomial equation solving and its application to computer vision. Licentiate thesis, Centre for Mathematical Sciences LTH, Lund University, Sweden, 2008.

[4] Martin Byröd, Zuzana Kukelova, Klas Josephson, Tomas Pajdla, and Kalle Åström. Fast and robust numerical solutions to minimal problems for cameras with radial distortion. In *Proc. Conf. Computer Vision and Pattern Recognition, Anchorage, USA*, 2008.

[5] R. I. Hartley and A. Zisserman. *Multiple View Geometry in Computer Vision*. Cambridge University Press, 2004. Second Edition.

[6] Hailin Jin. A three-point minimal solution for panoramic stitching with lens distortion. *Computer Vision and Pattern Recognition, 2008. CVPR 2008. IEEE Conference on*, pages 1–8, June 2008. ISSN 1063-6919.

[7] H. Stewénius. *Gröbner Basis Methods for Minimal Problems in Computer Vision*. PhD thesis, Lund University, April 2005.