Distance Dependent Lens Distortion Variation in 3D Measuring Systems Using Fringe Projection

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Abstract

Measuring systems using fringe projection provide the possibility of very accurate touchless measurements. For the measurement of small objects compact devices are possible. However, in the case of very close distances between the optical system and the measuring object there is a considerable influence of the measuring distance to the lens distortion. This will lead to considerable measuring errors if neglected. In typical cases of our measuring systems the amount of the distortion may change by a factor greater than two in the range of a distance difference of a few centimetres. Results of the distance dependence will be given for the lenses of two devices.

1 Introduction

It is well known that lens distortion correction is necessary in any systems used to perform measurements, e.g. in stereo image processing systems or in touchless 3D measuring systems based on fringe projection. Recently, measuring systems with short working distances are designed and produced in order to achieve a minimal size of the measuring device and to reduce the costs of the material. These systems will only work well, if the distortion will be reduced to a minimum. That means either to realise a distortion free optics which will be very expensive or to achieve a sufficient correction of the distortion. As it will be shown in this work, this is only possible if the change of the distortion depending on the distance will be considered in the case of short distances. As a short one should be noted distances smaller than about 30 times the focal length which has been derived empirically [9]. Consequently, a method to handle practicably the distance depending distortion should be developed.

Although it is known that different focus implies different distortion, it is usually neglected that if the focus is constant, the distortion differs, too, depending on the distance between the measuring object and the lens (measuring distance) in the range of a sharply mapped image. The reasons of the neglection of this fact are probably these two: first, the expected change of the distortion effects are assumed to be smaller than the measuring uncertainty, and, second, in practical applications it means often too high effort to determine the actual distance. Another reason may be the typical large measuring distances compared to the focal length of the lens. However, if the measuring distance is smaller than about 30 times of the focal length [9] the distance dependence of the lens distortion must not be neglected as it will be also shown in this work. Recently, a number of works have been

BMVC 2006 doi:10.5244/C.20.34

published concerning lens distortion determination and correction. Some standard works are for example [13, 14]. Usually, distortion is described by a distortion function including radial, decentering, and affine parameters [9] or as a distortion pattern using a grid model.

2 Variation of Lens Distortion

The knowledge of a distance or magnification dependence of lens distortion is older than 50 years. In 1955 Magill published his work about variation in distortion with magnification [10], where he mentioned the phenomenon of a changing amount of distortion depending on the working distance to the viewed object.

Let the magnification m_s for a distance s be defined as

$$m_s = \frac{f}{(s-f)} \tag{1}$$

where f is the focal length and denote by $M_s = 1/m_s$ the inverse magnification. In the following distortion should mean radial distortion if not other said.

Magill developed a formula [10] in order to calculate the distortion at any arbitrary distance or magnification which was extended by Brown [2] who stated a formula for the situation of known two radial distortion values Δr_{s_1} and Δr_{s_2} for the two distances s_1 and s_2 in order to predict the distortion value at any arbitrary focus distance s:

$$\Delta r_s = \alpha_s \Delta r_{s_1} + (1-\alpha_s)\,\Delta r_{s_2} \tag{2}$$

with

$$\alpha_s = \frac{s_2 - s}{s_2 - s_1} \cdot \frac{s_1 - f}{s - f}. \tag{3}$$

This formula was further modified and has been verified experimentally for radial and decentering distortion for certain conventional film cameras [2, 6]. Brown developed an extended model in order to describe the radial distortion variation outside the plane of best focus by a scaling factor $\gamma_{ss'}$

$$\Delta r_{ss'} = \frac{1}{\gamma_{ss'}} \Delta r_{s'} \tag{4}$$

where $\Delta r_{ss'}$ is the radial distortion at an object distance *s'* for a lens focussed at an object distance s and $\Delta r_{s'}$ the radial distortion at an object distance *s'* for a lens focussed at an object distance *s'*. The scaling factor $\Delta\gamma_{ss'}$ is given by

$$\gamma_{ss'} = \frac{s'(s-f)}{s(s'-f)}. \tag{5}$$

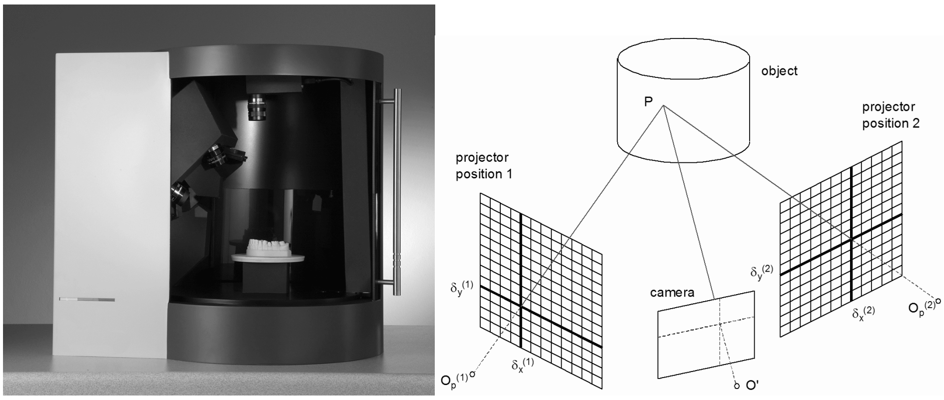
Fraser and Shortis [5] suggest the introduction of an empirically determined correction factor:

$$\Delta r_{ss'} = \Delta r_s + g_{ss'}(\Delta r_{s'} - \Delta r_s) \tag{6}$$

where $g_{ss'}$ is an empirically derived constant value and Δr_s the radial distortion at an object distance s for a lens focussed at an object distance s.

Another suggestion is given by Dold [4] who suggests a set of parameters which may be completely determined within a bundle adjustment process:

$$\Delta r_{dist} = \frac{1}{Z^*}\left[D_1 r'\left(r'^2 - r_0^2\right) + D_2 r'\left(r'^4 - r_0^4\right) + D_3 r'\left(r'^6 - r_0^6\right)\right] \tag{7}$$



(a) Device "kolibri flex 80" (b) Principle of phasogrammetry

Figure 1: Photograph of the device and measuring principle

where $Z^* \approx s$. For more details see [4, 8]. Brakhage introduces a method for the consideration of distance dependence of distortion effects for fringe projection systems with telecentric projection lenses using Zernike-polynomials [1].

In principal, for the description of lens distortion several different models are possible. The only important point is that the chosen model fits sufficiently the actual occurring distortion.

3 Situation, Model and Approach

3.1 Situation

Let us consider measuring systems using fringe projection based on the principle of phasogrammetry, e.g. the device “kolibri flex 80” (see Figure 1a) developed at the Fraunhofer IOF in Jena [7, 11]. The basic principle of phasogrammetry includes the projection of a fringe pattern from at least two different positions onto the measuring object whereas the second series is rotated 90° to the first one. Each of the fringe patterns consists of two series of pattern sequences, e.g. Gray-Code- and Cos-sequences, and produces phase values. The so called calibration camera remains in fix position according to the object and records the phase values. Thus every object point is characterised by at least four phase values. The phase values on the object points and their associated projection centres define spatial bundle of rays similar to those of the cameras in photogrammetry (see Figure 1b). The system “kolibri flex 80” is extended to a flexibly measuring system by adding two so called measuring cameras [7, 11] which contribute to the measuring values. Object points with a certain phase value correspond to the subpixeled camera co-ordinates. The complete system consists of one projector and three cameras.

Thus in order to calculate 3D co-ordinates of the measured object a triangulation between the corresponding pixels of the projector and the measuring cameras is performed. And here, unfortunately, the problem of distortion gets more importance than in common cases. This becomes clear when the data flow in the 3D measuring system is considered. See Figure 2 for illustration of the data flow.

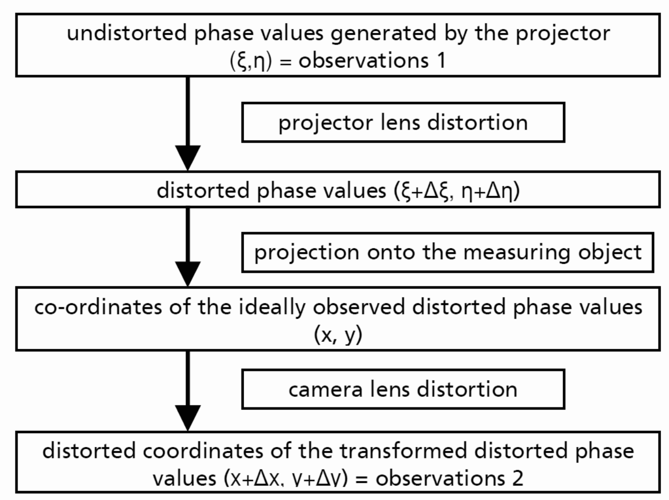


Figure 2: Scheme of the data flow in the systems using fringe projection including lens distortion

3.2 Model

Lens distortion describes the deviation of the actual vision ray between the observed object point and the point image in the image plane from the used camera model which describes the 3D-2D-mapping. Usually (see e.g. [9]) radial, decentering, and affine distortion $(\Delta x, \Delta y)$ is considered and determined, e.g. by

$$\begin{aligned} \Delta x &= x'\left(a_2 r'^2 + a_4 r'^4 + \ldots\right) + b_1\left(r'^2 + 2x'^2\right) + 2b_2 x'y' + c_1 x' + c_2 y'. \\ \Delta y &= y'\left(a_2 r'^2 + a_4 r'^4 + \ldots\right) + b_2\left(r'^2 + 2y'^2\right) + 2b_1 x'y' \end{aligned} \quad (8)$$

A more general model for the description of lens distortion is a matrix of distortion vectors in the image plane which enables the modelling of any arbitrary distortion. The distortion matrix is represented by a reduced number of distortion vectors usually arranged in a grid (see Figures 3 and 4). The amount of the distortion between the distortion grid points must be determined by interpolation, e.g. by bicubic spline interpolation.

This distortion matrix model can be extended easily to a model describing distance dependence. Now, we have a 3D matrix (cube), where the third dimension is the object distance which should cover the range of possible distances by meaningful representing layers.

Why distance dependence has to be considered in this kind of measuring devices? That is, first, because the absolute distance between projector and camera, respectively, and the measuring object is quite short, and, second, because there are considerable changes of the distance within the measuring volume due to the different tilt angles of the projector and the cameras.

3.3 Determination of the projector distortion

For the method proposed here a distortion free camera is necessary or a camera which is correctable with high precision. In a first step the distortion of this additional camera should be determined with high accuracy using any suitable method. We used the method described in [3]. Next a number of n phase images taken by the calibration camera are recorded by the device from different positions of the projector. The different projector positions are achieved by a rotation of the object table. The purpose of analysing n images is, first, to minimise the effect of erroneously recognising camera distortion as projector distortion, and, second, to reduce noise. As the minimum for n we took four images.

The procedure to estimate the projector distortion with given camera distortion having one phase image as input is the following. The resolution of the camera is decreased from C x R pixels to a grid with R/m x C/m rows and columns, respectively, by averaging m x m phase values. This reduces noise and closes gaps by undefined phase values. We obtain two lists of corresponding phase values $(\xi,\eta)_i$ and camera pixel values $(x+\Delta x, y+\Delta y)_i$. The observed pixel values already include the camera distortion. The given camera distortion $(\Delta x, \Delta y)_i$ is subtracted leading to $(x,y)_i = (x+\Delta x, y+\Delta y)_i - (\Delta x, \Delta y)_i$. A projective 2D-2D transform T is calculated in such a way that

$$(T\,(\xi,\eta)_i - (x,y)_i)^2 \rightarrow min. \tag{9}$$

After inversion of T obtaining T^{-1} the projector distortion is now estimated as

$$(\Delta\xi,\Delta\eta)_i = T^{-1}\,(x,y)_i - (\xi,\eta)_i \tag{10}$$

for every averaged phase point $(\xi,\eta)_i$. Now, a grid in the phase image plane must be defined using a number of grid points $(\xi,\eta)_j$ with a meaningful distance. The calculated distortion vectors $(\Delta\xi,\Delta\eta)_i$ are now inserted into the grid points $(\xi,\eta)_j$ using a meaningful interpolation.

The determination of the distortion will be performed for a certain constant distance $dist_1$ getting the distortion matrix for that certain distance. Now the whole procedure must be performed again for another distances $dist_2$ to $dist_n$ until the whole range of possible distance is covered.

3.4 Determination of the camera distortion

The distortion of the cameras is performed analogously to the projector distortion determination using the corrected projector to produce a calibration pattern. In particular, a distortion matrix for the camera is produced for one certain constant distance. This will be realised by a perpendicular adjustment of the optical axis of the camera to the plane on which the calibration pattern will be projected. In the “kolibri flex 80” device this can be realised using the position of the calibration camera which looks directly down to the rotation plate (see Figure 1a). Alternatively, the camera can be removed and the distortion determination is performed outside the device.

For the determination of one 2D layer of the 3D distortion matrix, n phase images taken by the camera. Now the determination of the distortion of the camera is performed in the same way as described in section 3.3 for the projector distortion determination. For a more detailed description of the procedure see [3].

3.5 Distance dependent distortion correction

The determined 3D distortion matrices are used directly to construct a correction operator which manipulates the calculated image co-ordinates, i.e. the phase values as those of the projector and the camera pixel co-ordinates, respectively (see [7, 11]). However, to apply the distance dependent operator the distance between the object and the optics must be known. That is, of course, not the case because the distance will be a result of the measurement. That's why the distance dependent distortion correction requires a rough estimation of the distance in a first step before the exact 3D measurement is possible. A rough estimation with an uncertainty of some millimetres is sufficient. If the distance of every image point is estimated the operator can be applied producing the corrected value. The corrected values are the input for the bundle adjustment process which will be performed in the usual way [7, 8, 11].

4 Results

The experiments to determine the distance dependent variation of lens distortion were performed as described in the previous section. We examined two prototypes KF1 and KF2 of the device "kolibri flex 80". Distortion was determined for the projection unit (including lens ApoComponon 40) and two observation cameras with lenses Cinegon 1.4/8 (SK8) and Cinegon 1.4/12 (SK12 - all lenses by Schneider-Kreuznach). The optical systems also include mirrors for ray reflection which may contribute to the resulting distortion. The image resolution of the projector chip was 1024 x 768 (pixel size: 20 μm) and of the cameras 1392 x 1040 (pixel size: 4.65 μm) pixels. The considerable changes concerning the distances of the optical components to the measuring volume between these two prototypes are due to a new adjustment.

Table 1 shows the results for the distortion determination of the projection unit of devices KF1 and KF2 for the determined layers. The value *mdist* denotes the distance of the main ray from the projection centre to the centroid of the measuring volume, and the value *md* describes the mean length of the distortion vectors. The Figures 3 and 4 show the distortion matrices of the different layers for KF1 and KF2. The *are* (average residual error) value characterises the mean resulting error of the image co-ordinates:

$$are = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\left(x_i - x_i^{id}\right)^2 + \left(y_i - y_i^{id}\right)^2}. \qquad (11)$$

where $\left(x^{id}, y^{id}\right)_i$ are the (estimated) ideal image co-ordinates.

The resulting distortion matrices were analysed concerning the possibility of a functional description of the distortion. For this the software BINGO (see also [8]) and the methods described in [3] were used. The distortion of the cameras was easy to describe

device	KF1			KF2		
layer	*mdist* (mm)	*md* (pixel)	*are* (pixel)	*mdist* (mm)	*md* (pixel)	*are* (pixel)
1	263	0.16	0.06	332	0.08	0.02
2	249	0.17	0.03	309	0.08	0.02
3	235	0.20	0.04	287	0.10	0.03
4	223	0.23	0.05	265	0.14	0.03
5	212	0.30	0.07	243	0.20	0.05

Table 1: Distance depending distortion of projection unit, devices KF1 and KF2

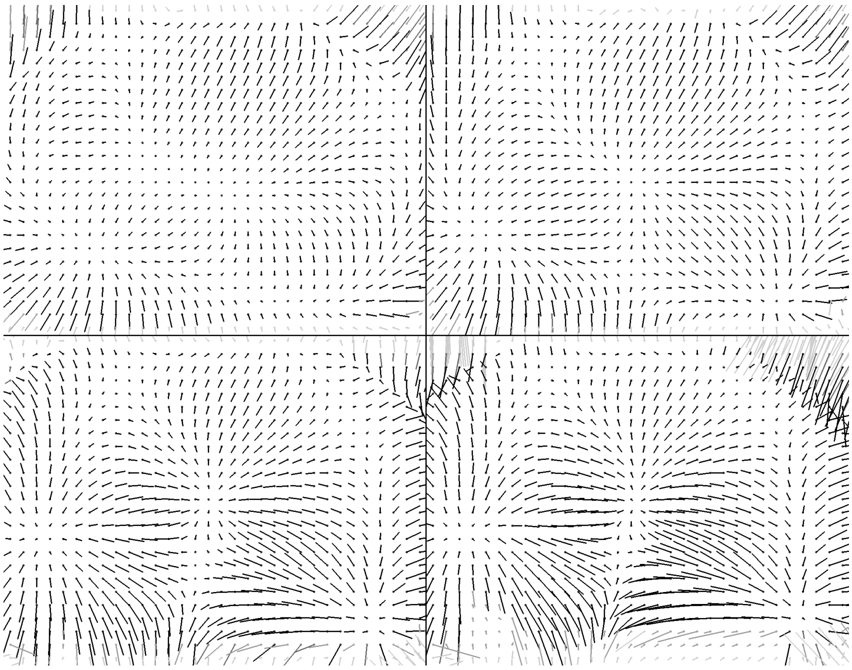


Figure 3: Four of the five different determined layers of distortion of projection unit, device KF1. Upper line: layer1 (left), layer2 (right), lower line: layer4 (left) and layer5 (right). The distortion vectors are scaled by a factor of 100 according to the grid size.

by a simple model of pure radial distortion using three parameters (symmetry centre of distortion (x_0, y_0) and one distortion coefficient d_2) according to

$$\Delta x_{rad} = (x - x_0)\left(\frac{r'}{r} - 1\right); \Delta y_{rad} = (y - y_0)\left(\frac{r'}{r} - 1\right); r = \frac{r'}{1 + d_0 + d_2 r'^2}. \qquad (12)$$

The parameter d_0 in (12) has a 100% correlation with the camera constant c and realises the second zero position of the distortion curve (see Figures 5 and 6). Hence d_0 is no proper distortion coefficient and it is omitted in the documentation of Table 2. The results of device KF2 were similar to those of KF1, because two lenses of the same type were used. That's why the documentation is left out here.

lens	SK8 (KF1)			SK12 (KF1)		
layer	*mdist* (mm)	$d_2 \cdot 10^{-8}$ (pixel)	*are* (pixel)	*mdist* (mm)	$d_2 \cdot 10^{-8}$ (pixel)	*are* (pixel)
1	230	-2.20	0.04	230	-1.51	0.08
2	205	-2.62	0.04	-	-	-
3	180	-2.86	0.04	180	-1.84	0.08
4	155	-3.25	0.03	155	-1.93	0.08
5	130	-3.63	0.04	-	-	-

Table 2: Distance dependent distortion of cameras SK8 and SK12, device KF1

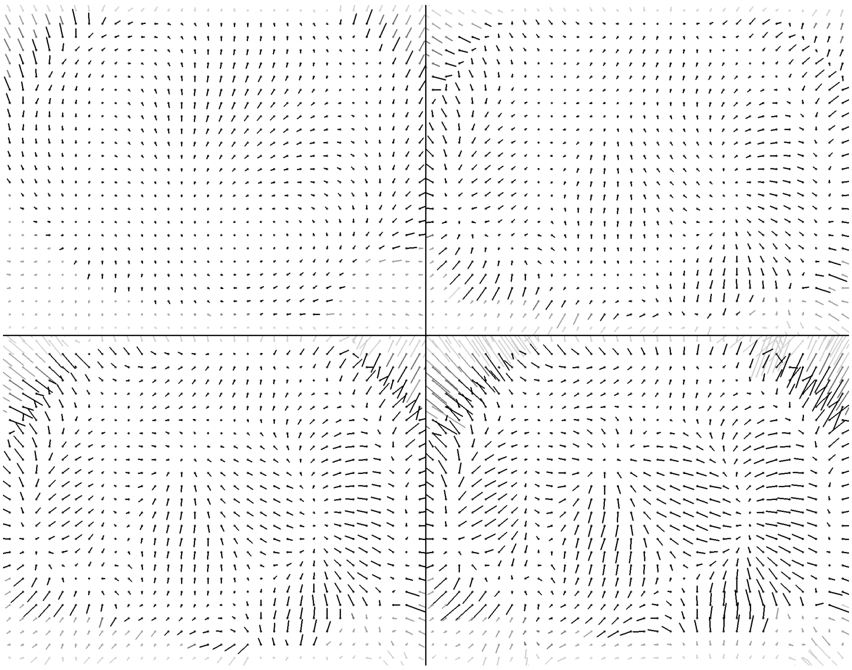


Figure 4: Four of the five different determined layers of distortion of projection unit, device KF2. Upper line: layer1 (left), layer3 (right), lower line: layer4 (left) and layer5 (right). The distortion vectors are scaled by a factor of 100 according to the grid size

Contrary, the distortion of the projection unit could not be described sufficiently exact by a functional description even using up to 30 parameters [8].

5 Measurement Example

In order to validate the determined distortion matrices and the realised correction operators a plane surface was measured by the KF1 device using three different modes. The first one is without correction, the second one with conventional correction not using 3D distortion variation information and the third one realising the whole correction. The measuring object was the plane surface of a coated mirror with a measured area of 70 mm diameter. For illustration see Figure 6. The difference between the highest and the lowest measured plane point (PV value) decreased from 22 to 7 μm.

6 Discussion and Outlook

The results show that distortion strongly changes for close distances. In our experiments we observed an increasing of the radial distortion of the two wide angle camera lenses between 1.2 (SK12) and 1.65 (SK8) in the considered distance range and a change of the distortion of the projector lenses between 1.9 (KF1) and 2.5 (KF2). Obviously, if not corrected these distortion effects lead to considerable errors in the measurement process.

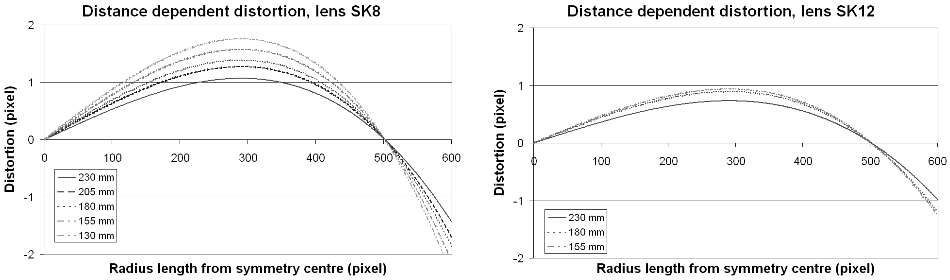


Figure 5: Distance dependent distortion curves of the lenses SK8 and SK12

As our examples show modern and future applications will use the range of close distances with inverse magnifications less than 30 contrary to the assumptions of Shortis et al. [12]. It should be concluded that the distance dependent distortion must be concerned in high precision optical measurement systems in order to avoid considerable measuring errors. Future work should deal with further analysis of distortion variation sources (aperture, sharpness) and the comparison of lenses of the same type concerning their distortion variation behaviour.

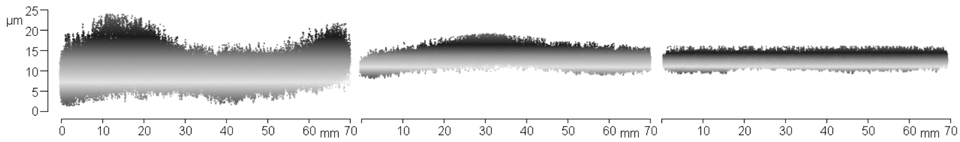


Figure 6: Reconstruction of a plane surface with 70 mm diameter; uncorrected (PV=22μm), conventionally corrected (PV=10μm), and distance dependent corrected (PV=7μm). The Y-axis is inflated by a factor of 1000

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