Multi-Modal Face Image Super-Resolutions in Tensor Space

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Abstract

Face images of non-frontal views under poor illumination with low resolution reduce dramatically face recognition accuracy. To overcome these problems, super-resolution techniques can be exploited. In this paper, we present a Bayesian framework to perform multi-modal (such as variations in viewpoint and illumination) face image super-resolutions in tensor space. Given a single modal low-resolution face image, we benefit from the multiple factor interactions of training tensor, and super-resolve its high-resolution reconstructions across different modalities. Instead of performing pixel-domain super-resolutions, we reconstruct the high-resolution face images by computing a maximum likelihood identity parameter vector in high-resolution tensor space. Experiments show promising results of multi-view and multi-illumination face image super-resolutions respectively.

1 Introduction

Super-resolution aims to generate higher resolution images given a single or a set of multiple low-resolution input images. The computation of super-resolution requires the recovering of lost high-frequency information occurring during the image formation process. Super-resolution can be performed using either reconstruction-based [4, 5, 6, 7] or learning-based [10, 8, 9, 11, 12, 13] approaches. In this work, we focus on learning-based approaches.

Capel and Zisserman [11] used eigenface from a training face database as model prior to constrain and super-resolve low-resolution face images. A similar method was proposed by Baker and Kanade [8]. They established the prior based on a set of training face images pixel by pixel using Gaussian, Laplacian and feature pyramids. Freeman and Pasztor [10] took a different approach for learning-based super-resolution. Specifically, they tried to recover the lost high-frequency information from low-level image primitives, which were learnt from several general training images. A very similar image hallucination approach was also introduced in [13]. They used the primal sketch as the prior to recover the smoothed high-frequency information. Liu and Shum [12] combined the PCA model-based approach and Freeman’s image primitive technique. They developed a mixture model combining a global parametric model called “global face image” carrying common facial properties, and a local nonparametric model called “local feature image”
recording local individualities. The high-resolution face image was naturally a composition of both.

To go beyond the current super-resolution techniques which only consider face images under fixed imaging conditions in terms of pose, expression and illumination, we present in this work a Bayesian model to perform multi-modal face image super-resolutions in tensor space. Given a single modal low-resolution face image, we benefit from the multiple factor interactions of training tensor, and super-resolve its high-resolution reconstructions across different modalities. Instead of performing pixel-domain super-resolutions, we reconstruct the high-resolution face images by computing a maximum likelihood identity parameter vector in high-resolution tensor space.

The paper is organized as follows. Section 2 introduces multilinear analysis and tensor singular value decomposition (SVD). In section 3, we derive a Bayesian framework to perform multi-modal super-resolutions, and present an algorithm optimizing the high-resolution identity parameter vector in tensor space. Section 4 discusses experimental results before conclusions are drawn in section 5.

2 Multilinear Analysis: Tensor SVD

Multilinear analysis [1, 3, 2] is a general extension of the traditional linear methods such as PCA or matrix SVD. Instead of modelling the relations within vectors or matrices, multilinear analysis provides a means to investigate the mappings between multiple factor spaces. In this context, the multilinear equivalents of vectors (first order) and matrices (second order) are called tensors, multidimensional matrices or multiway arrays. Tensor singular value decomposition or higher-order singular value decomposition (HOSVD) [3] is a multilinear generalization of the concept of matrix SVD. In the following, we denote scalars by lower-case letters ($a, b, \ldots; \alpha, \beta, \ldots$), vectors by upper-case ($A, B, \ldots$), matrices by bold upper-case ($A, B, \ldots$), and tensors by calligraphic letters ($A, B, \ldots$).

Given an $N^{th}$-order tensor $A \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$, an element of $A$ is denoted as $A_{i_1 \ldots i_N}$ or $a_{i_1 \ldots i_N}$, where $1 \leq i_n \leq I_n$. If we refer to $I_n$ rank in tensor terminology, we generalize the matrix definition and call column vectors of matrices as mode-1 vectors and row vectors of matrices as mode-2 vectors. The mode-$n$ vectors of the $N^{th}$ order tensor are the $I_n$-dimensional vectors obtained from $A$ by varying index $i_n$ while keeping the other indices fixed. We can unfold or flatten the tensor $A$ by taking the mode-$n$ vectors as the column vectors of matrix $A_{(n)} \in \mathbb{R}^{I_{(n)} \times (I_1 I_2 \ldots I_{n-1} I_{n+1} \ldots I_N)}$. These tensor unfoldings provide an easy manipulation in tensor algebra and if necessary, we can reconstruct the tensor by an inverse process of mode-$n$ unfolding.

We can generalize the product of two matrices to the product of a tensor and a matrix. The mode-$n$ product of a tensor $A \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ by a matrix $M \in \mathbb{R}^{J_1 \times I_n}$, denoted by $A \times_n M$, is a tensor $B \in \mathbb{R}^{I_1 \times \cdots \times I_{n-1} \times J_n \times I_{n+1} \times \cdots \times I_N}$ whose entries are computed by

$$(A \times_n M)_{i_1 \ldots i_{n-1} j_n i_{n+1} \ldots i_N} = \sum_{i_n} a_{i_1 \ldots i_{n-1} i_n i_{n+1} \ldots i_N} m_{j_n i_n}.$$  

This mode-$n$ product of tensor and matrix can be expressed in terms of unfolding matrices for ease of usage,

$$B_{(n)} = MA_{(n)}.$$  

(1)
Given the tensor $A \in R^{I_1 \times I_2 \times \cdots \times I_N}$ and the matrices $F \in R^{J_n \times I_n}$ and $G \in R^{J_m \times I_m}$, the following property holds true in tensor algebra [2, 3]:

$$(A \times_n F) \times_m G = (A \times_m G) \times_n F = A \times_n F \times_m G.$$  

In singular value decompositions of matrices, a matrix $D$ is decomposed as $U_1 \Sigma U_2^T$, the product of an orthogonal column space represented by the left matrix $U_1 \in R^{J_1 \times J_1}$, a diagonal singular value matrix $\Sigma \in R^{J_1 \times J_2}$, and an orthogonal row space represented by the right matrix $U_2 \in R^{J_2 \times J_2}$. This matrix product can also be written in terms of mode-$n$ product as $D = \Sigma \times_1 U_1 \times_2 U_2$. We can generalize the SVD of matrices to multilinear higher-order SVD (HOSVD). An $N^{th}$-order tensor $A \in R^{I_1 \times I_2 \times \cdots \times I_N}$ can be written as the product

$$A = Z \times_1 U_1 \times_2 U_2 \times \cdots \times_N U_N,$$  

(2)

where $U_n$ is a unitary matrix, and $Z$ is the core tensor having the property of all-orthogonality, that is, two subtensors $Z_{\alpha} = \alpha$ and $Z_{\beta} = \beta$ are orthogonal for all possible values of $n$, $\alpha$ and $\beta$ subject to $\alpha \neq \beta$. The HOSVD of a given tensor $A$ can be computed as follows. The mode-$n$ singular matrix $U_n$ can directly be found as the left singular matrix of the mode-$n$ matrix unfolding of $A$, afterwards, based on the product of tensor and matrix as in Eq.(1), the core tensor $Z$ can be computed by

$$Z = A \times_1 U_1^T \times_2 U_2^T \times \cdots \times_N U_N^T.$$  

Eq.(2) gives the basic representation of multilinear model. If we investigate the mode-$n$ unfolding and folding, and rearrange Eq.(2), we can have

$$S = B \times_n V_n^T,$$

where $S$ is a subtensor of $A$ corresponding to a fixed row vector $V_n^T$ of the singular matrix $U_n$, and

$$B = Z \times_1 U_1 \cdots \times_{n-1} U_{n-1} \times_{n+1} U_{n+1} \cdots \times_N U_N.$$  

This expression is the basis for recovering original data from tensor structure. If we index into basis tensor $B$ for more particular $V_n^T$, we can get different modal sample vector data.

## 3 Multi-Modal Super-Resolutions in Tensor Space

In this section, we first build a tensor structure for face images of different modalities including varying illumination, viewpoint (head pose) and people identity. We then derive an algorithm for super-resolution in tensor parameter vector space.

### 3.1 Modelling Face Images in Tensor Space

We construct a tensor structure from multi-modal face images and use HOSVD to decompose them. The decomposed model can be expressed as

$$D = Z \times_1 U_{idens} \times_2 U_{views} \times_3 U_{illums} \times_4 U_{pixels},$$
where tensor $D$ groups the multi-modal face images into a tensor structure, and the core tensor $Z$ governs the interactions between the 4 mode factors. The mode matrix $U_{\text{idens}}$ spans the parameter space of different people identities, the mode matrix $U_{\text{views}}$ spans the parameter space of changing head poses, and the mode matrix $U_{\text{illums}}$ spanning the space of varying illumination parameters, the mode matrix $U_{\text{pixels}}$ spanning space of face images.

With decomposed tensor of multi-modal face images, we can perform super-resolution in tensor parameter vector space. In such a formulation, the observation is an identity parameter vector computed by projecting testing low-resolution face images onto a tensor constructed from low-resolution training images, and proposed algorithm super-resolve the true identity parameter vector in a tensor constructed from high-resolution training images. We start with the pixel-domain image observation model. Assuming $D_L$ is a vectorized observed low-resolution image, $D_H$ is the unknown true scene, and $A$ is a linear operator that incorporates the motion, blurring and downsampling processes, the observation model can be expressed as

$$D_L = AD_H + n,$$

where $n$ represents the noise in these processes.

Figure 1: An illustration of our multi-modal super-resolution process in tensor space using a multi-view super-resolution example.

The unknown high-resolution image $D_H$ and observed image $D_L$ have identity parameter vectors that lie in the respective tensor spaces. These parameter vectors provide a unique representation for any people identity independent of the potentially varying modalities such as viewpoint and illumination. Rather than performing super-resolution on pixel-domain modal by modal, we derive a model for the reconstruction of identity parameter vectors in the high-resolution tensor space.

Based on the tensor algebra introduced in section 2, suppose we have a basis tensor

$$B = Z \times_2 U_{\text{views}} \times_3 U_{\text{illums}} \times_4 U_{\text{pixels}},$$

we can index into this basis tensor for a particular viewpoint $v$ and illumination $l$ to yield a basis subtensor

$$B_{v,l} = Z \times_4 U_{\text{pixels}} \times_2 V_v^T \times_3 V_l^T,$$
for each of the face imaging modalities. Then the subtensor containing the individual image data can be expressed as

\[ D_{v,l} = B_{v,l} \times_1 V^T + \mathcal{E}_{v,l}, \]  

(5)

where \( V^T \) represents the identity parameter row vector and \( \mathcal{E}_{v,l} \) stands for the tensor modelling error for modalities of viewpoints \( v \) and illumination \( l \). For ease of notation and readability, we will use the mode-1 unfolding matrix to represent tensors. Then the matrix representation of Eq.(5) becomes

\[ D^{(1)}_{v,l} = V^T B^{(1)}_{v,l} + e_{v,l}, \]  

(6)

The counterpart of pixel-domain image observation model (3) is then given as

\[ \hat{B}^T v,l \hat{V} + \hat{e}_{v,l} = \hat{B}^T v,l V + \hat{A} e_{v,l} + n, \]  

(7)

where \( \hat{B}^T v,l \) and \( \hat{B}^T v,l \) are the low-resolution and high-resolution unfolded basis subtensors, \( \hat{V} \) and \( V \) are the identity parameter vectors for the low-resolution testing face image and unknown high-resolution image.

Independent of changing viewpoints \( v \) and illuminations \( l \), the low- and high-resolution parameter vectors \( \hat{V} \) and \( V \) are the unique representations of the low-resolution input and its corresponding high-resolution image to be estimated. Without loss of generality we can rewrite Eq.(7) as

\[ \hat{B}^T v,l \hat{V} + \hat{e}_{v,l} = \hat{B}^T v,l V + \hat{A} e_{v,l} + n, \]  

(8)

where \( \hat{B}^T v,l \) and \( \hat{B}^T v,l \) are the unfolded basis tensors, and \( \hat{E} \) and \( E \) are the combined tensor modelling error over all modal face images.

Low-resolution observation images contain very little high-frequency information after the processes of downsampling and blurring, so we can safely neglect the error \( \hat{E} \) and multiply both sides of Eq.(8) by \( \Psi = (\hat{B}^T v,l (\hat{B}^T v,l)^{-1}) \) on the left, we obtain

\[ \hat{V} = \Psi \hat{B}^T v,l V + \Psi \hat{E} + \Psi n, \]  

(9)

where \( \Psi \) is the pseudoinverse of \( \hat{B}^T v,l \). Eq.(9) gives the relation between the unknown “true” identity parameter vector \( \hat{V} \) and the observed low-resolution counterpart \( \hat{V} \). In Fig.(1), we use the multi-view example to illustrate the whole process of our multi-modal super-resolutions in tensor space.

### 3.2 A Bayesian Formulation

We use the Bayesian estimation algorithm to solve Eq.(9). The maximum a posteriori probability (MAP) estimation of the high-resolution identity parameter vector \( V \) can be expressed as

\[ \hat{V} = \arg \max_V \{ p(\hat{V}|V)p(V) \}, \]  

(10)

where \( p(\hat{V}|V) \) is the conditional probability modelling the relations between \( \hat{V} \) and \( V \), and \( p(V) \) is a prior probability. We can assume the prior probability as Gaussian

\[ p(V) = \frac{1}{Z} \exp(-(V - \mu_V)^T \Lambda^{-1}(V - \mu_V)), \]
where $\Lambda$ is the covariance matrix for all the training parameter vectors $V_i$. In our tensor structure, the identity parameter vectors $V_i$ comes from the row vectors of orthogonal matrix $U_{idens}$. In this sense, the prior $p(V)$ just simply leads the optimum $\hat{V}$ in Eq.(10) to the mean value $\mu_V$. So Eq.(10) degenerates to the maximum likelihood (ML) estimator

$$\hat{V} = \arg \max_V p(\hat{V}|V). \quad (11)$$

To solve the above equation, we define a total noise $F$ that consists of the tensor representation error $E$ and the pixel-domain observation noise $N$, and rewrite Eq.(9) as

$$\hat{V} = \Psi AB^T V + \Psi F. \quad (12)$$

Now we need derive the distribution of the projected noise $p(\Psi F)$. Before that, we can write the probability distribution of $F$ as

$$p(F) = \frac{1}{Z} \exp \left( -(F - \mu_F) K^{-1} (F - \mu_F) \right),$$

where $K$ is a defined diagonal covariance matrix and $Z$ is a normalization constant. Since $B^{(1)} B^{T(1)}$ is nonsingular, $p(\Psi F)$ can also be modeled as jointly Gaussian, then we have

$$p(\Psi F) = \frac{1}{Z} \exp \left( -(\Psi F - \Psi \mu_F)^T Q^{-1} (\Psi F - \Psi \mu_F) \right), \quad (13)$$

where $\Psi \mu_F$ is the projected mean error and $Q$ is the new covariance matrix computed by

$$Q = \Psi K B^{T(1)}. \quad (14)$$

Based on Eq.(12) and Eq.(13), we find the conditional probability $p(\hat{V}|V)$ as

$$p(\hat{V}|V) = \frac{1}{Z} \exp \left( - (\hat{V} - \Psi AB^T V - \Psi \mu_F)^T Q^{-1} (\hat{V} - \Psi AB^T V - \Psi \mu_F) \right).$$

Then finally we obtain the ML estimator $\hat{V}$ as

$$\hat{V} = \arg \min_{\hat{V}} \left( (\hat{V} - \Psi AB^T V - \Psi \mu_F)^T Q^{-1} (\hat{V} - \Psi AB^T V - \Psi \mu_F) \right). \quad (15)$$

In the above expression of ML estimation, the statistics of mean $\mu_F$ and covariance matrix $K$ can be computed based on the training images. Assuming we have $I$ training people, and for each of them we have $M$ training images of different modalities, then we estimate the mean and covariance matrix as follows

$$\mu_F \cong \frac{1}{IM} \sum_{i=1}^I \sum_{m=1}^M (D_{i,m}^{T(1)} - AB^T V_i),$$

and

$$K \cong \frac{1}{IM} \sum_{i=1}^I \sum_{m=1}^M (D_{i,m}^{T(1)} - AB^T V_i - \mu_F)(D_{i,m}^{T(1)} - AB^T V_i - \mu_F)^T.$$
where $\hat{D}_{v,l}$ represents every low-resolution training image and $V_i$ is the high-resolution identity parameter vector for each training people. We set off-diagonals of $K$ to zero and use Eq.(14) to obtain $Q$.

We use the iterative steepest descent method for ML estimation of $\tilde{V}$. Defining $C(V)$ as the cost function to be minimized, $V$ can be updated in the direction of the negative gradient of $C(V)$. The updating equation can be expressed as

$$V_{n+1} = V_n - \alpha \nabla C(V_n),$$

(16)

where $\alpha$ is the step size. We choose the cost function according to Eq.(15) as

$$C(V) = (\hat{V} - \Psi \hat{A}\hat{B}^T(1)V - \Psi \mu_F)^T Q^{-1} (\hat{V} - \Psi \hat{A}\hat{B}^T(1)V - \Psi \mu_F),$$

and take the derivative of $C(V)$ with respect to $V$, the gradient can be computed as

$$\nabla C(V) = -\hat{B}^{(1)} A^T \Psi^T Q^{-1} (\hat{V} - \Psi \hat{A}\hat{B}^T(1)V - \Psi \mu_F).$$

In summary, everything but $\hat{V}$ and $V$ are known (In our experiments, the low-resolution images are blurred and downsampled manually, so we keep the the image observation model parameter $A$ in the data preparation processes). The identity parameter vector $\hat{V}$ on low-resolution tensor space is obtained by projecting the testing face image $\hat{D}$ onto basis subtensors of all modalities, and then reconstruct them by projecting back, the parameter vector that gives the minimum reconstruction error is chosen as $\hat{V}$, which is essentially a modal estimation process. Based on Eq.(6), the expression can be written as

$$\hat{V} = \arg \min_{V_{v,l}} \| \hat{D} - \hat{B}_{v,l}^{(1)} \hat{V}_{v,l} \|,$$

(17)

for all the combinations of viewpoints $v$ and illumination $l$, where $\hat{V}_{v,l}$ can be computed as $\hat{V}_{v,l} = \Psi_{v,l} \hat{D}$ and $\Psi_{v,l}$ is the pseudoinverse of $\hat{B}_{v,l}^{(1)}$. To summarize, the complete algorithm is as follows.

- Compute the initial estimate of $V_0$ by bilinearly interpolating the given low-resolution testing face image to the same size of the high-resolution training images, and projecting it onto the training tensor space.
- Obtain the identity parameter vector $\hat{V}$ using Eq.(17).
- Repeat the process of optimizing $V_n$ in Eq.(16).
- Obtain the ML estimation $\tilde{V}$.

4 Experiments

In this section, we firstly present results on super-resolving face images in multiple views given a single view low-resolution testing image. We then show results on super-resolving face images under different illumination conditions given a single illumination low-resolution testing image.
Figure 2: Example images in our dataset: (a), (b), (c), (d) and (e) are 56 × 36 face images at frontal, yaw -/+45 degrees and tilt -/+ 45 degrees views; (f), (g) and (h) are 56 × 36 face images under three different illumination conditions of Illum-I, Illum-II and Illum-III.

Figure 3: Experiments on super-resolving multi-view face images given a single view low-resolution input: (a) are low-resolution input images (14 × 9) at different single views (obtained by downsampling original testing input images); (b) - (f) are high-resolution reconstruction results (56 × 36) at frontal, yaw -/+45 degrees, and tilt -/+45 degrees views respectively; and (g) - (k) are ground truth face images at these 5 views.

For our experiments, we used face images from a subset of AR, FERET and Yale databases to form two datasets for multi-view and multi-illumination experiments respectively. The multi-view dataset has a set of 1475 face images of 295 different individuals,
in which each individual has 5 different view face images. For multi-illumination dataset, we have a set of 399 images of 133 persons, each of them have 3 face images with 3 different illuminations (Illum-I, Illum-II and Illum-III). Originally face images from AR, FERET and YALE databases have different sizes, and also the area of the image occupied by face varies considerably. To establish a standard training dataset, we aligned these face images manually by hand marking the location of 3 points: the centers of the eyeballs and the lower tip of the nose. These 3 points define an affine warp, which was used to warp the images into a canonical form. Examples of our dataset are shown in Fig.2.

We performed two sets of experiments on multi-modal super-resolutions using our model derived in Section 3. In the first experiment, we used our multi-view dataset. Given a low-resolution single view face image, we super-resolved 5 high-resolution outputs at 5 different views covering the frontal, yaw -/+45 degrees, and tilt -/+45 degrees. Some example results from this experiment are shown in Figure 3. In the second experiment, we used our multi-illumination dataset to perform super-resolution and yield three high-resolution outputs under three different illumination conditions (Illum-I, Illum-II and Illum-III) given only one single illumination low-resolution input. Some example results are shown in Figure 4. In both of these two experiments, we used the “leave-one-out” methodology. That is in each of the dataset, those images which were not selected as the testing image were used to construct the model tensors.

The high-resolution reconstruction results shown in Fig.3 and Fig.4 are clearly promising and go beyond what existing methods are capable of in terms of generalizing into significantly different views and illuminations in super-resolution. Although not perfect, it does provide the potentials to improve the recognition performance based on the super-resolved high-resolution multi-modal face images.

![Figure 4: Experiments on super-resolving face images under multiple illumination conditions given a single illumination low-resolution input: (a) are low-resolution input images (14 x 9) under 3 different illumination conditions (obtained by downsampling original testing input images); (b) - (d) are high-resolution reconstruction results (56 x 36) at Illum-I, Illum-II and Illum-III respectively; and (e) - (g) are ground truth face images under these 3 illumination conditions.](image-url)
5 Conclusion

In summary, we present a multi-modal face image super-resolution system in tensor space. By introducing the tensor structure that models multiple factor interactions into a Bayesian framework, we can super-resolve the high-resolution tensor identity parameter vector, given a single modal low-resolution face image. Based on the super-resolved identity parameter vector, we can reconstruct the multiple high-resolution face images across different views and under changing illumination conditions. Experimental results verify our declaration.

We have not conducted the face recognition experiments yet. In the future work, based on the super-resolved identity parameter vector in high-resolution tensor space, we will directly perform face recognition across different views and under changing illumination conditions without the reconstructions of multi-modal face images.

References


