An efficient method of using explicit shape models of objects in boundary instantiation is to apply one dimensional edge searches in locations where boundaries are likely to occur. In many important cases, linear edge operators produce at best only weak responses. We investigate here the use of three different statistical measures applied over a sliding ‘dipole’ as candidates for detecting weak boundaries. Their performance is compared with an implementation of the Canny operator as a benchmark on synthetic images of step edges in random noise and on certain difficult real images. In the former case their performance compares favourably with the Canny operator, while in the latter case they can produce significant responses where the Canny operator detects only weakly or not at all.

INTRODUCTION

In most applications of computer vision and image processing, the correct location of the boundaries, between different objects, or between object and background is of central importance in achieving a correct image interpretation. The literature abounds with methods for detecting these boundaries, which make use either of the different properties of the regions on either side of the boundary, or the fact that the boundary is characterized by a pronounced grey-level discontinuity or edge.

The edge based approach is much favoured in interpretation of unconstrained three dimensional scenes, where the properties of regions may not easily be predicted. Region based approaches are often used in cases when the image is more constrained, and may be considered to be two dimensional, e.g. in remote sensing or microscopy. Both approaches are based on models of the world which are acknowledged to be flawed. Region properties tend to be less well-defined near the very boundaries they are used to detect, and edges are often weaker on true boundaries than at other, semantically irrelevant, points. Both region and boundary methods tend to be applied without reference to high level knowledge concerning the likely location and properties of boundaries.

Recent experience in our group has shown that considerable improvements in boundary detection can be made by directed, one-dimensional edge detection. The direction comes from a model of what is expected in the image, providing a prediction of the positions and orientations of expected boundaries. The exact boundary locations are determined by one dimensional edge searches across the predicted boundary. The confidence in a detected edge point can be assessed by reference to local and global models of the expected edge. This approach has produced very encouraging results in application fields as disparate as industrial inspection and histology of muscle sections. In order to make this type of analysis applicable to a wide range of applications, we require a robust boundary cue locator which operates by one dimensional search, avoids the problem of ill defined edges and which does not depend critically on the nature of the boundary. In this paper we describe some operators which approach this requirement by measuring properties of the distribution of image values on either side of the boundary. We show that using this approach, boundary detection performance can be as good as or better than optimal methods of edge primitive detection in terms of sensitivity and accuracy, while allowing the flexibility of being adapted to local models of the image.

BOUNDARY DETECTION OPERATORS

Given a prediction of where to look for a boundary, its correct position is located by searching along a line perpendicular to its putative orientation. To increase signal to noise ratio, it is best to integrate the response across some width perpendicular to this line. The search therefore takes place within an elongated rectangle, and we are seeking a partition of the rectangle along its length which produces the two most distinct distributions of image values. In order to make appropriate comparison of the distributions of image values on either side of the boundary, it is important that equivalent areas are sampled. It is also necessary to avoid confusion due to the inclusion in the sampling of nearby boundaries with other regions. The detector we have used is a “dipole” consisting of a rectangular box partitioned...
into two poles, whose length and width can be varied according to the grey level and geometrical model of the expected edge. This dipole is scanned across the edge; at each point on the scan the distributions in either pole are sampled and compared in ways described below.

Three different statistics have been implemented for the comparison of the two poles: the entropy, the standard deviation and the mean of the distributions.

**Figure 1.** The entropy dipole response to a perfect step edge. The dipole (in this case of half length 10 pixels) is scanned across the window at the top (width 30 pixels). The mark at the top of the window indicates the true edge position ("best"), and that at the bottom the edge position located by the dipole ("found"). The entropy value in pole A ($E_A$) as it crosses the edge is shown in trace a, that of pole B ($E_B$) in trace b, and that of the whole dipole ($E_t$) in trace c. Trace d shows the response of the operator measure $2 \times E_t - (E_A + E_B)$.

**Entropy**

The entropy of a probability density function is given by $E = -\sum p_i \ln(p_i)$ where $p_i$ is the probability of occurrence of state $i$. It has frequently been used as a threshold selection measure in region-based segmentation, where its usefulness lies in the fact that it acts as a measure of "peakiness" or compactness of a distribution. A very narrow distribution of states gives a low value for $E$, whereas as broad distribution of roughly equally populated states gives a high value. When a distribution is being divided into two distributions on either side of a threshold, for example, the division which minimises the sum of the two entropies produces the intuitively optimal result. In our case we are dividing a distribution not by a threshold, but by spatially partitioning the area from which it is sampled. However, the principle of producing the most compact distributions from the region on either side of the partition is still a useful one.

Figure 1 shows the behaviour of $E_t$, $E_A$ and $E_B$ as the dipole is scanned across a simple step edge, where $E_A$ and $E_B$ are the entropies in poles A and B, and $E_t$ is the total entropy in the window. Fortunately, entropy is a self-normalising measure. The entropies in each of the poles rises as that pole crosses the edge; that is $E_A$ has a maximum on the right hand side of the edge, $E_B$ has a maximum on the left hand side of the edge. Both have low values (0 in this ideal case) when the partition is on the edge. $E_t$ on the other hand has a maximum when the partition is on the edge. We calculate the signal $2 \times E_t - (E_A + E_B)$ which rises sharply to a maximum on the edge. Figure 2a shows the response of the entropy dipole to a noisy edge.

**Figure 2.** The response of the three dipole operators to a step edge in gaussian noise. The edge amplitude is 1 grey level and the noise standard deviation is 7 grey levels. Trace a is the entropy response, trace b is the SD, and trace c the significance of means. Trace d is a density profile along the window integrated across its width.

**Standard Deviation**

The entropy measure responds to the shape of the distribution but is costly to calculate. A cheap alternative, which also responds to the shape of the distribution and which is also self normalising, is the standard deviation. In similar vein to the entropy, the measure is $2 \times SD_t - (SD_A + SD_B)$ and has a similar response.
Mean

One way of looking at our approach is to say that we are examining two distributions to determine whether they appear significantly different. A straightforward method of doing this is to examine the significance of the difference in means given by

\[ z = \frac{|\mu_A - \mu_B|}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}} \]

Figure 2 shows the responses of each of these three operators in locating a step edge with superimposed noise in a case where the image signal to noise ratio is low (0.14). All of these operators show the capability of locating step edges in noise.

PERFORMANCE

To determine which of these operators has the best properties of sensitivity and accuracy, we have undertaken a systematic test of their responses to a step edge in noise, varying the step size, noise standard deviation and dipole width and length. As a benchmark by which the responses could be measured we included the response of the Canny edge detector in the test.

The Canny Operator

The edge operator due to Canny is widely regarded as the best compromise between sensitivity and accuracy in the detection of edge primitives. Indeed it was designed to provide the optimum response to a step edge amongst gaussian noise. It consists in essence of a one dimensional gaussian smoothing of the raw image in the direction parallel to the edge, followed by a one dimensional derivative of gaussian convolution across the edge. The widths of the gaussians in the two directions are typically equal, and the edge response is integrated across some sampling width. Canny's implementation provides for detection of edges at different scales and combination of the responses at different scales to produce an edge map. Our requirement is not for an edge primitive detector, but an edge locator. The different scales at which the Canny operator can be applied correspond roughly to the varying dipole size of our detectors. We do not need to track the response to the edge through scale space, we are merely interested in the sensitivity and localisation accuracy at a particular scale.

The Test

The images used consisted of 256 x 256 pixels with a single vertical step edge extending the height of the image, on which had been superimposed gaussian random noise. The signal to noise ratio (edge amplitude divided by the noise standard deviation) was varied from 0.14 to 1.33. The dipole widths and (half) lengths were varied from 10 to 50 pixels in steps of 10. In the case of the Canny operator gaussian smoothing of standard deviation 1, 3, 5, 7 and 9 pixels was applied.

For each point in parameter space 10 measurements of edge position were made at completely separate positions along the edge. (Thus for some of the broader supports more than one noise image was required.) The measurement consisted of scanning the dipole across the whole width of the image and measuring the responses of the dipole and Canny operators. In the case of the Canny operator the output of the smoothed one dimensional derivative was integrated along the edge direction. In all cases, the detected edge position was taken to be the position of absolute maximum response.

The measurements made on each scan were:

- The distance of the located edge from the true edge.
- The response at the position of the true edge.
- The response at positions distant from the true edge.

From these measures at each point in parameter space we have:

- A (sparse) histogram of localisation error.
- A distribution of response signal.
- A distribution of response noise.

From which we derive:

- Sensitivity : (signal mean - noise mean)/(noise standard deviation)
- Localisation accuracy : The mean localisation error
- Localisation precision : The standard deviation of the localisation error.

Results

Derived performance values were obtained over a range of dipole widths and half-lengths. In each case the Canny smoothing standard deviation used as an equivalent to the detector length is such that two s.d.'s is about equal to the dipole half-length. The two cases are not directly comparable in terms of the contributions of their support regions, and the decision to adopt a particular combination of support sizes as being equivalent is a fairly subjective one. The two standard deviation cut off was selected, since the gaussian weighting is certainly significant within this boundary, and indeed for some distance beyond it. The Canny results are included to give some idea of the scale of values.

Figures 3 to 5 show examples of some results at a particular scale. They show how the derived values...
vary with the signal to noise ratio of the image using a
dipole width of 30 pixels and a half length of 20. An
s.d. of 9 was used for the corresponding Canny
operator.

![Figure 3](image3.png)

Figure 3. Sensitivity of the dipole operators compared
with the Canny operator as a function of the image
signal to noise ratio (edge amplitude divided by noise
s.d.). The dipole width and edge integration width is
30, the dipole half length is 20 and the s.d. of the
smoothing gaussian in the Canny case is 9.

Figure 3 shows the variation in sensitivity of the
different operators. Not surprisingly, the sensitivity of
all the operators increases steadily with the signal to
noise ratio of the image. Against the Canny
benchmark, the performance of the entropy dipole is
poor, particularly at low signal to noise values. The
significance of means dipole is better at low signal to
noise, having about 60% of the Canny response. The
SD dipole has very similar sensitivity to Canny at very
low signal to noise ratios, becoming increasingly
better as the signal to noise ratio increases above
0.25.

![Figure 4](image4.png)

Figure 4. Edge localisation accuracy (mean error) for
the 30 x 20 (9) support. (See figure 3). Off scale
values at low image signal to noise are not shown.

![Figure 5](image5.png)

Figure 5. Edge localisation precision (standard
deviation of the error) for the 30 x 20 (9) support. (See
figure 3). Off scale values at low image signal to noise
are not shown.
Figures 4 and 5 show the variation of localisation accuracy and precision using the 20 x 30 pixel support. Both accuracy and precision give a measure of how reliably the edge is located, and the graphs show similar behaviour. Both give good values (< 1 pixel), down to some signal to noise threshold at which the localisation becomes quickly unreliable. In the case of the entropy dipole, the threshold is rather higher than for Canny. SD and significance of means dipoles have slightly higher thresholds than Canny. Notice that this threshold can occur at values of sensitivity which appear fairly high.

With smaller support sizes, similar behaviour is observed. All the sensitivities are reduced, of course, and the threshold at which localisation accuracy becomes unreliable is higher. The sensitivity of the SD dipole at a signal to noise ratio in the image of unity, using a 10 x 10 pixel support, is about twice that of the Canny operator, compared to about five times as in figure 3.

Real Images

Experiments with test images provide confidence that the dipole operators are likely to be reasonable candidates for providing boundary cues. The model of a step edge among random noise, however, is not an ideal one for the cases in which we would like to apply these operators, namely to diffuse or weak edges among structured noise. Experiments with a number of images, particularly of biological material, indicate that one or other of the dipole operators can give a strong response at faint or noisy boundaries where the Canny operator responds only weakly or not at all. Obviously cases can be found in which these operators fail to detect a boundary, but in such cases the Canny operator also fails. There is no clearly best candidate among the three dipole operators. The significance of means response is consistently similar to that of the SD operator, and consistently more noisy, making it clearly the worst. Whether the best results are obtained by the entropy or SD operator depends on the image in question, notwithstanding the poor showing of the former on the noise images. The difficulty in modeling real cases means that it is difficult to make an objective assessment of performance or to demonstrate power in boundary location. Further study may allow us to find methods of determining the most appropriate operator for particular cases. For illustrative purposes we present some examples of edge responses.

Figure 6. Detection of a weak edge in a chromosome image. The search region is indicated by the bracketed window.

a - Entropy dipole response
b - SD dipole response
c - Significance of means dipole response
d - Canny response
e - Projected density profile along the search line

The arrow indicates the edge position determined from a. The dipole width is 7 pixels and half length is 10 pixels. The corresponding standard deviation for the Canny operator is 5.

Figure 6 shows a search for a difficult edge in a chromosome image. A shape model predicts the existence of a boundary in a certain direction. The search is confused by the existence of strong edges in addition to the weak true edge. The responses of all four operators under test are shown. The dipole operators, including the entropy dipole give significant responses while the Canny operator produces no response. Figure 7 is part of a radiograph of a hip prosthesis. The required boundary is that between the bone and the retaining cement. The edge in this case can be very indistinct, but dipole search with a large support can provide important cues to its position. No case has been observed in which a boundary which can be detected by the Canny operator cannot be detected by one or more of the dipole operators.
It is important to notice that the image signal to noise ratio at the edge is quite high in both of these cases: about 2.3 for the chromosome image and about 1.9 for the radiograph. Even if we choose to model the boundary detection using a step edge amongst noise, our working region in real images is likely to be well to the right of, or beyond, the scale of figures 3, 4 and 5.

DISCUSSION

The approach taken in designing the dipole operators described here is that something is known about the location and orientation of a boundary between two regions and that its true position can be determined by measuring some statistic of the distribution of image values on either side of the edge. De Sousa\(^4\) has described a similar application of sliding statistical tests to radiographs and natural texture images. One of his measures was identical to the significance of means dipole described here, which has consistently shown behaviour similar to that of the SD dipole, but more noisy. Several authors\(^5,6\) have considered using a comparison of medians or other order statistics to detect edges. These methods are all used in the context of edge preserving smoothing to reduce impulse noise. We did not consider this to be an appropriate model for the type of boundary detection we wish to achieve.

Three statistics whose properties seem reasonable for the task have been implemented and tested systematically on an artificial image, with the Canny operator acting as a benchmark. This test of sensitivity and accuracy was an exacting one since the Canny operator is optimised in one sense for the detection of step edges among gaussian noise. The performance of the entropy dipole was disappointing, but that of the significance of means dipole was better. The SD dipole gave encouraging results, being about as accurate as the Canny operator and in many cases much more sensitive.

The application of the dipole detectors to difficult real-world images has shown that one or more of them can provide useful boundary cues in cases where linear edge detection fails – the very cases in which model based instantiation is most necessary. Despite its poor showing in detecting model step edges, the entropy dipole appears to retain some promise as a boundary cue operator in real-world images.

REFERENCES


